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Phase Angle Calculations: Considerations and Use Cases

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Introduction

This paper addresses the calculation and use of phase angles reported from Phasor Measurement Units (PMUs). PMUs report phase angles, referenced to a common time source such as GPS, resolved to the interval (+180, -180] degrees. The phase angle changes as the measured waveform deviates from a reference cosine signal, driven by changes in grid frequency. Applications and tools must manage the reported phase angles such that these phase angles can be used for engineering analysis and real-time tools. This paper lays out the considerations that users should be aware of when handling phase angles, including:

1. Phase angle mathematics and background;
2. The phasor angle reported from a PMU;
3. The phase angle difference calculation problem statement;
4. Definition of phase angle wrapping and unwrapping;
5. Phase angle difference calculation methodologies;
6. Practical considerations for phase angle difference calculations;
7. Introduction of accumulated phase angle;
8. Description of Roll Over Counter (ROC) and Angle Distance; and
9. Actual implementations of phase angle difference calculations.

The considerations described above and calculation of phase angle differences plays an important role in a number of practical applications. These applications depend on and use the phase angle measurements for providing information to real-time tools and offline engineering applications, including:

- **Event Detection:** Phase angle differences can be used for event detection algorithms; sudden changes in phase angle differences can identify events on the bulk power system such as switching of transmission or tripping of transmission, generation resources, or large blocks of load.
- **Visualization and Contouring:** Time trends, polar charts, and topological maps can visualize angle differences across the grid. This information provides operators and engineers with indications of system stress where large angle gradients are cause by transfers or weakened topology.
- **Phase Angle Limit Monitoring:** Phase angle limit monitoring includes providing operator situational awareness, additional security assessment measures, and operating procedures for mitigating excessive phase angle differences.
- **Oscillation Monitoring:** Modal analysis techniques for oscillation monitoring and oscillation detection use phase angle differences as inputs since the angle differences are closely related to the electromechanical behavior of the interconnected machines synchronized to the grid.
- **Islanding Detection:** Island detection uses unwrapped phase angle differences referenced to a given phase angle in the grid to identify unsynchronized portions of the system that split away from the "ribbon" of angle differences due to lack of synchronism.
- **Remedial Action Schemes (RAS):** Phase angles and angle differences can be used in RAS to detect undamped or poorly damped oscillatory behavior or large angle swings caused by a large disturbance on the grid.

The goal of this paper is to provide background and guidance to industry practitioners implementing phase angle applications.

Phase Angle Formulation

A Phasor Measurement Unit (PMU) reports measured values time-tagged to UTC[1]. These measured parameters are the amplitude of the signal, x, the frequency, *f*, the rate-of-change of frequency, ROCOF, and the phase angle, ϕ. This paper is particularly concerned with the phase angle, which is the measured phase of the power system relative to a reference signal[2] synchronized to UTC.

The power system operates approximately at a nominal frequency (50 or 60 Hz). However, system frequency is constantly changing as the balance between load and generation changes. The signal could be modeled as the signal *x(t)*:

$$x(t) = X_m \cos[\omega t + \emptyset] \quad (1)$$

where the amplitude $X_m(t)$, the frequency is ω, the phase is Φ and all are functions of time. The phasor measurement unit (PMU) is designed to report these values, typically many times per second, in accordance with IEEE Standard C37.118-1 (2011). The values of the amplitude and the phase are named "synchrophasor." The frequency, and a parameter identified as the rate of change of frequency, are also reported.

As an example, consider the case where the system frequency f(t) is constant and off-nominal. If the frequency is less than nominal the period will be longer than the reference and the sinusoid will gradually move away from the reference, as seen in Figure 1. (The figures that follow are "synthetic." They show greatly exaggerated frequency differences for the purpose of illustration. They also incorporate abrupt changes in frequency, something that cannot happen on a power system.)

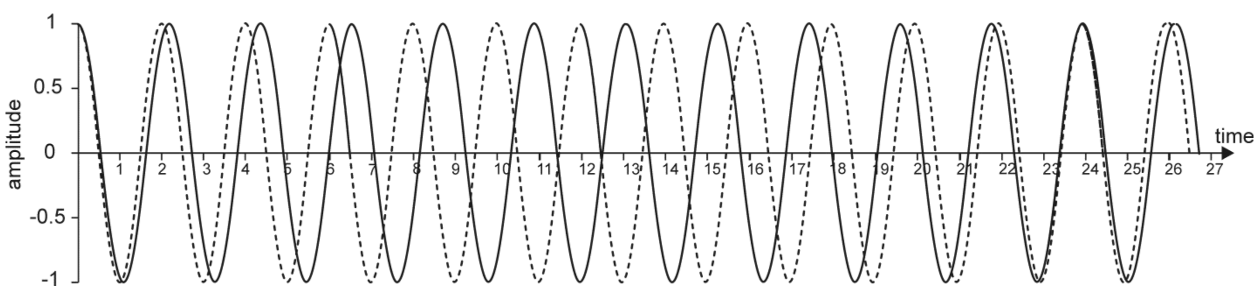

Figure 1. Cosine waveforms, with phase = 0 at t = 0, solid line at lower frequency

If the relative phase angle generated by Equation (1) is plotted as a function of time, it appears as a straight line as seen in Figure 2.

[1] Universal Coordinated Time, often provided by a Global Positioning System (GPS) satellite receiver and clock.

[2] The reference signal used to derive the power system phase has a constant frequency while the power system does not, therefore enabling the derivation of a relative phase angle to reference UTC time.

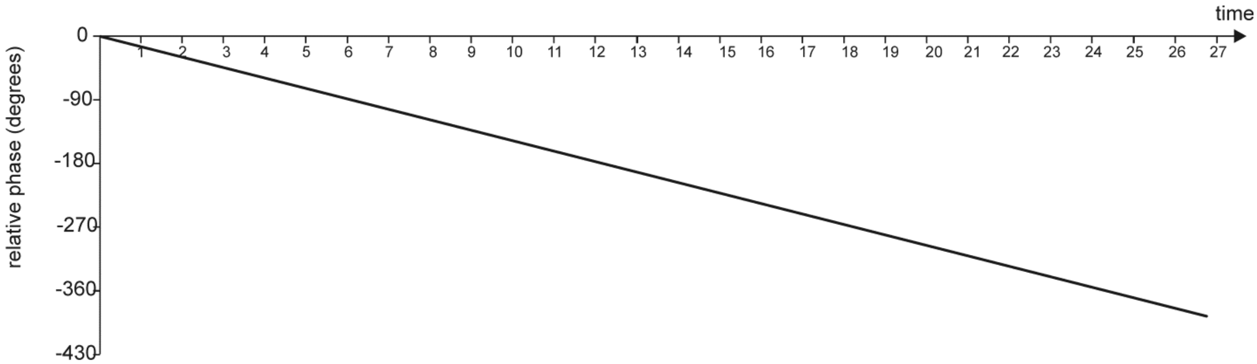

Figure 2. Phase evolution for Figure 1

If the data were plotted for a longer time, the relative phase would continue to accumulate. However, the graph does not represent the results as reported by a phasor measurement unit.

The PMU examines the signal for a relatively short time, a duration known as a measurement window. This time duration is just a few cycles long, much shorter than the time indicated. The phase indicated in Figure 2 is zero at time zero, and 360° at time 24. For the PMU, with a time window of a few cycles, these situations cannot be distinguished, and the PMU reports the angle as zero at both times. For a PMU with a two-cycle window, the situation for both zero time and time 24 is as seen in Figure 3.

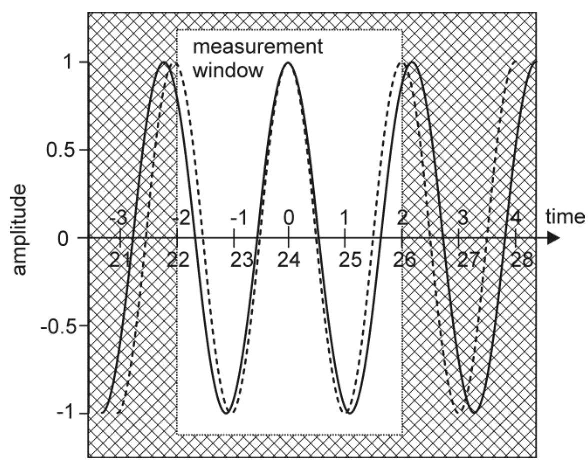

Figure 3. Two-cycle measurement window, centered at t =0 and t = 24

Figure 3 shows that, with a short window of observation, there is no difference between the signals at time=0 and time=24. At each of these instants, the signal is in-phase with the reference. Based on this information, the phase is reported by the PMU as phase = zero. The fact that the PMU can give only values for the phase that are between -180° and +180° means that the phase as reported by the PMU does not match the total reality. A PMU reporting every cycle would give the values indicated by dots in Figure 4.

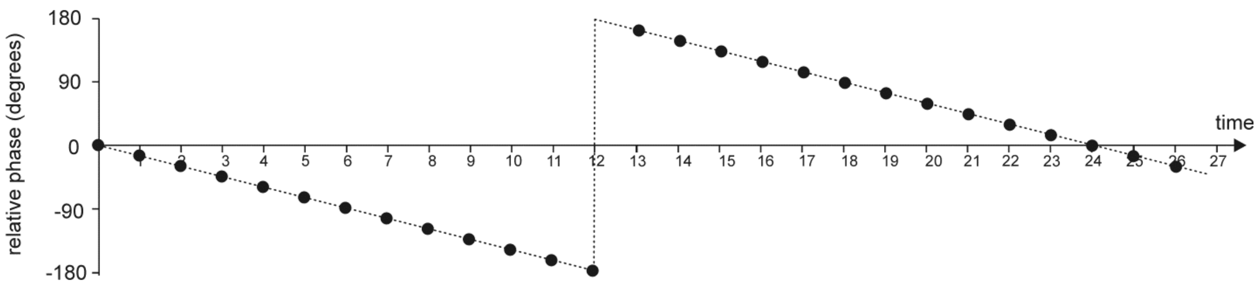

Figure 4. PMU values corresponding to Figure 2

The slope of the line in Figure 4 is the same as that of the line in Figure 2, but the angle appears to “wrap” at time 12, when the angle reaches -180°. But note that the wrapping is not something the PMU does. Viewing the signal it measures through a short time window, it correctly gives the temporally local phase. It has no knowledge of wrapping: that is something evident only with a longer-term view.

The situation near |180°| is interesting because although the values that are near to 180° do not accumulate beyond that number, they can nevertheless be used to calculate what the total phase is. Since the appearance of wrapping does not occur always in the same direction, the matter is not simple.

If the frequency is not constant, the rate at which phase accumulates will vary. In Figure 5, the frequency is first lower than the reference, but at time=6 the frequency suddenly becomes the same as the reference, and that continues until time=18, when the frequency is increased above the reference value.

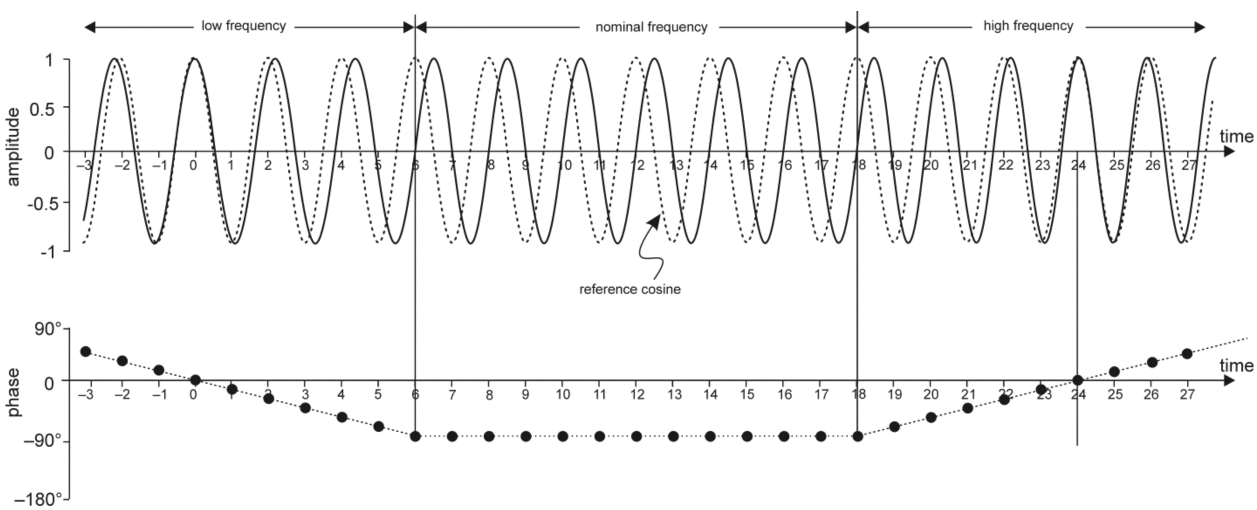

Figure 5. Slow system followed by fast system giving zero phase at time zero and time 24

In this example, the total phase accumulated between points t=0 and t=24 is zero. In general, the total is path-dependent, and cannot be determined without considering the complete interval of interest.

The process of finding the accumulated phase is therefore one that must distinguish between the situations shown in Figure 2 and the “phase” portion of Figure 5. Both have a phase value of zero at time t=0 and t=24. But only the situation in Figure 2 has accumulated a phase of 360°. If the circumstances shown in Figure 5 were allowed to continue, the phase would appear to wrap at time = 36, when the accumulated phase passed through +180° and was reported as a negative phase difference.

To summarize, phase angle is determined relative to a reference cosine at the nominal system frequency and synchronized to UTC. It is a continuous unbounded function of time. Synchrophasors are estimated over a short window of the waveform and the phase angle can only be known by its principal value between –180° and +180°. "Wraps" of the phase angle past ±180° are observed in the longer-term reported data. The number of wraps that occur from some set time can be calculated from these time-series observations.

Definition of Phase Angles from PMUs

The phase angle reported by a PMU is defined as the principal value (PV) of the signal relative to a cosine wave at the nominal system frequency and synchronized to UTC. This process characterizes the measurement as follows:

- The reference cosine wave will have a positive maximum value at the rollover of the UTC second.
- The time reference for any synchrophasor calculation can start at *any* UTC second rollover. This is based on the assumption of periodicity mentioned above.
- If the power system is exactly at the nominal system frequency, the phase angle of the signal will be constant from measurement to measurement, and will be the angle offset from the reference cosine wave.
- If the power system frequency is lower than the nominal system frequency, the relative phase angle will appear to decrease from measurement to measurement. In Figure 2, the solid line has the lower frequency. The relative phase is *defined* as negative. On a phasor diagram drawn for the reference frequency, the rotation would be clockwise, and the rate of rotation will be the difference between the nominal and actual frequency: $\Delta f = f_{\text{system}} - f_0$.[3]
- The characteristics for off-nominal frequencies described above will be somewhat different if the off-nominal frequency varies with time (i.e., it is not constant).

Phase Angle Difference Issues (Problem Statement)

Phase angles based on an absolute time reference $\phi\ (t)$ are not particularly useful in power system analysis. Instead, for most purposes the principal value $\phi\ (t \bmod T)$ is used. While it is not necessary to keep track of a particularly long time t, it is still important that the angles are all measured at the same time instant and to the same reference. This means angle differences *within* the power system can be calculated from the difference between the measured angles at two points. Using a time synchronized reference signal has solved the problem of determining system phase angle differences over a wide area.

The synchrophasor angle is a continuous function that can increase or decrease. While it would be possible for a PMU to add (or subtract) 360° every time the PV of the phase changes by about

[3] Some PMUs make the measurement of the frequency of the incoming signal by essentially demodulating the signal down to baseband as if it were an FM signal. The frequency thus obtained is then used in the equation $f_{\text{system}} = \Delta f + f_0$. Note that method can result in a negative value for the demodulated frequency internal to the PMU.

that amount, this is not done. The reported phase angles are kept manageable, and represented as a number between–180° and + 180° degrees; however, this creates a problem when comparing two measured phase angles within the power system because the true angle between two points could be on opposite sides of a 180° point reported by the PMU. Figure 6 shows two angles where the PV does not wrap around ± 180° and the arithmetic angle difference calculation is accurate. Figure 7 illustrates the issue where the PV of one angle measurement wraps around ± 180° followed by the other angle wrapping. This causes numerical issues that must be accounted for; this is the topic of this paper.

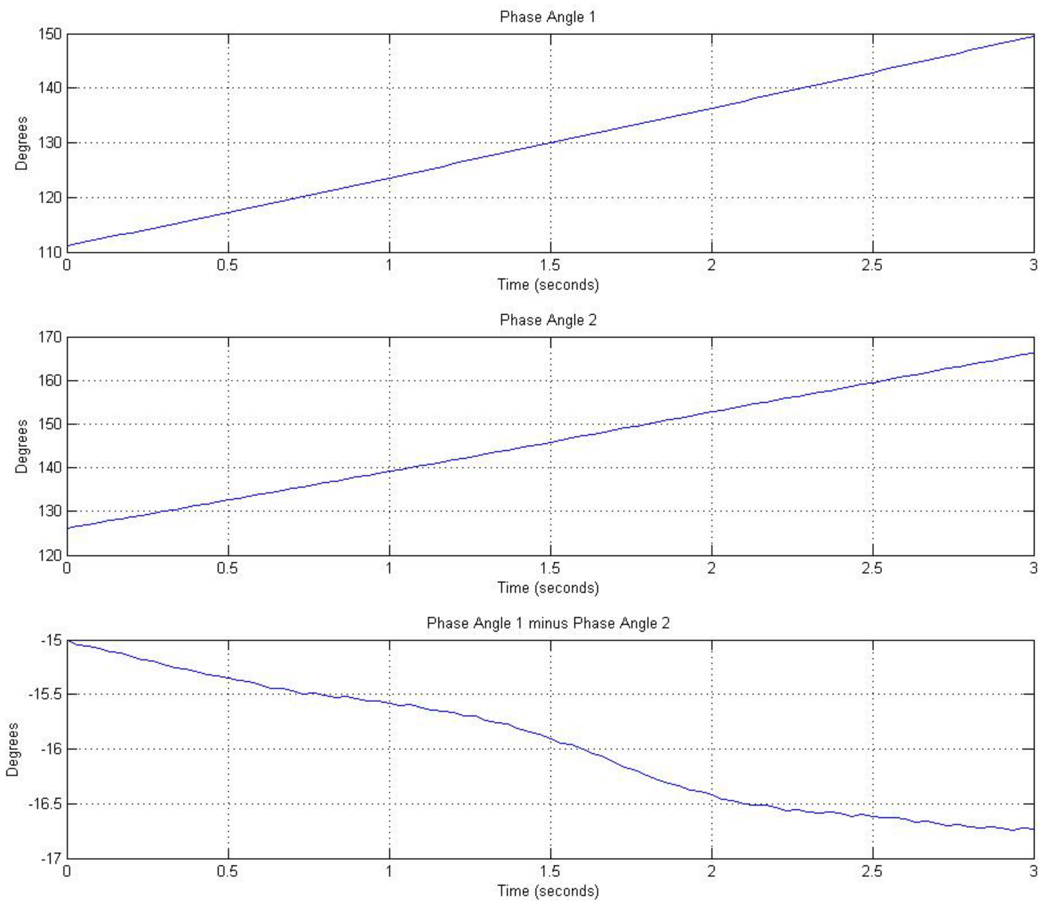

Figure 6. Example of Phase Angle Difference Calculation with No Wrap Around ± 180°

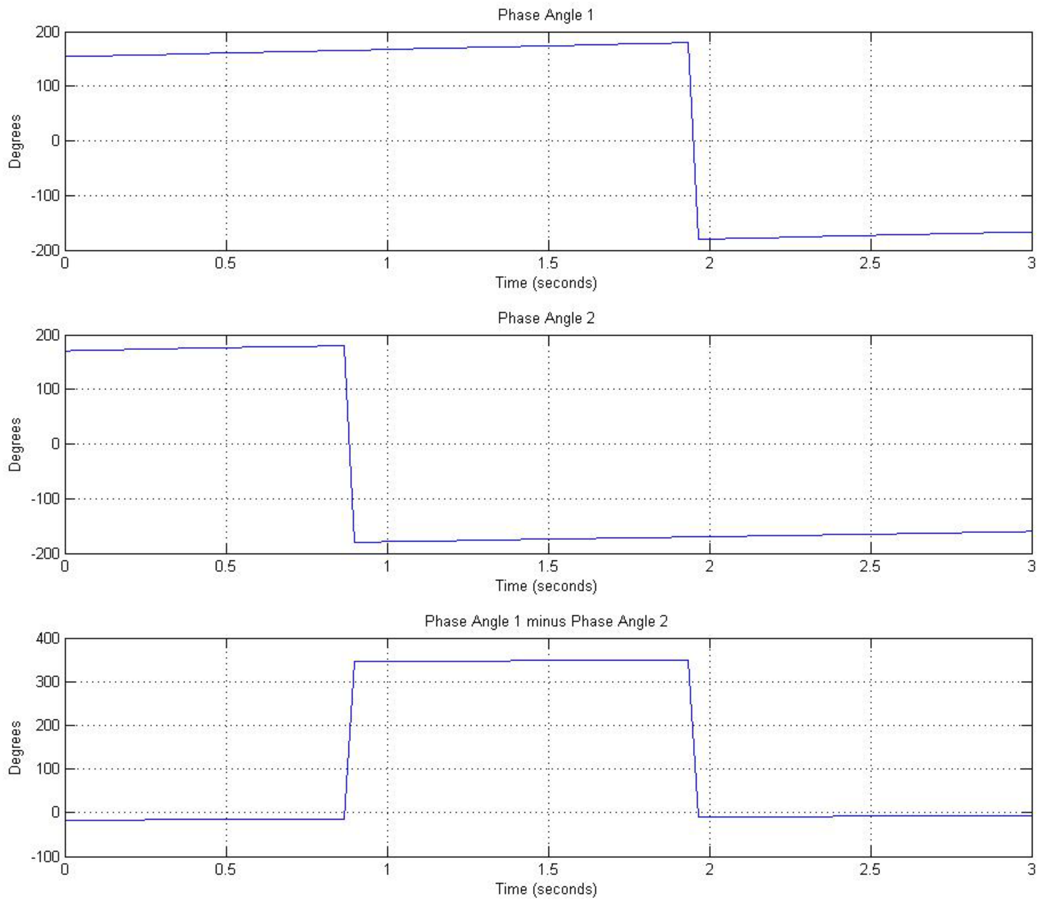

Figure 7. Example of Issue with Phase Angle Difference Calculation with Wrap Around ± 180°

Definition of Angle Wrapping

Suppose the wrapping operation is defined by the operator Ω with $\Omega\colon R \rightarrow (-\pi, \pi]$. Then, Ω can be defined as

$$\Omega(\varphi) = \begin{cases} \varphi - 2n\pi, & \text{where } n \in I \text{ is an integer so that } -\pi < \varphi - 2n\pi < \pi \\ \pi, & \text{where } n \in I \text{ is an integer so that } \varphi - 2n\pi = \pi \end{cases} \qquad (2)$$

In other words, the wrapping operator maps the principal value of any phase angle measured in radians to a unique value between –π and π. Any time the principal value of the angle ϕ goes over π, the wrapped angle $\Omega(\varphi)$ will wrap around to –π. Similarly, when the principal value of ϕ goes below –π, the wrapped angle $\Omega(\varphi)$ will switch to π. This is illustrated in Figure 8 which shows an unwrapped angle φ and the corresponding wrapped angle after the operation of $\Omega(\varphi)$.

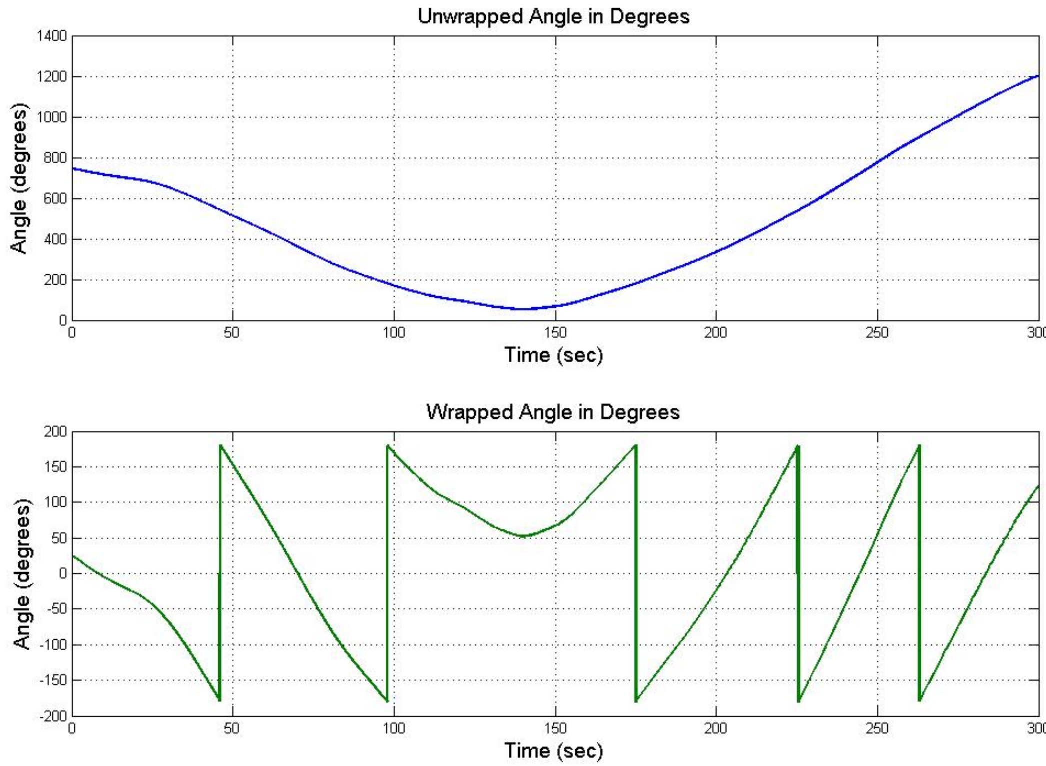

Figure 8. Wrapped and Unwrapped Angles

Definition of Angle Unwrapping

Consider a time series of phase angle measurements from a PMU denoted $\phi(t_k)$, where t_k varies from t_0 onwards in increments of the sampling time Δt. As noted above, the angle $\phi(t_k)$ lies within the range (-π,π]. Unwrapping is defined as the process of "reversing" the wrapping operation introduced in the previous subsection. Mathematically, it can be defined by an operator Ψ with $\Psi: (-\pi, \pi]\ X\ I \rightarrow R$, where Ψ can be defined recursively as follows:

$$\begin{cases} \Psi\big(\phi(t_0)\big) = \phi(t_0), \\ \Psi\big(\phi(t_k)\big) = \phi(t_k) + \Psi\big(\phi(t_{k-1})\big) - \phi(t_{k-1}) + 2n\pi, \text{where } n \text{ is an integer} \\ \qquad\qquad\qquad \text{that minimizes } |\phi(t_k) - \phi(t_{k-1}) + 2n\pi|,\ \forall\, k \neq 0. \end{cases} \quad (3)$$

Basically, the unwrapping operator adds an angle offset -2π or 2π to subsequent wrapped angle values whenever it detects a large change in the wrapped angle value. As an example, let us say that the sampling frequency is 30 Hz, and two consecutive angle values are ϕ(10) = -3.14 rad, and ϕ(10.333) = 3.14 rad respectively. This implies that there was an angle wrap-around or rollover between the time t = 10 sec and t = 10.333 sec. Accordingly, the value of *n* that minimizes the absolute value $|\phi(t_k) - \phi(t_{k-1}) + 2n\pi| = |3.14 - (-3.14) + 2n\pi|$ is, n = -1 so that $|3.14 - (-3.14) - 2\pi| = 0.0032$ rad. Therefore, the angle offset is incremented by -2π in (3) for t > 10. When the operator Ψ is applied repeatedly, the wrapped angle measurements as reported by PMUs can be unwrapped as shown in Figure 9 below.

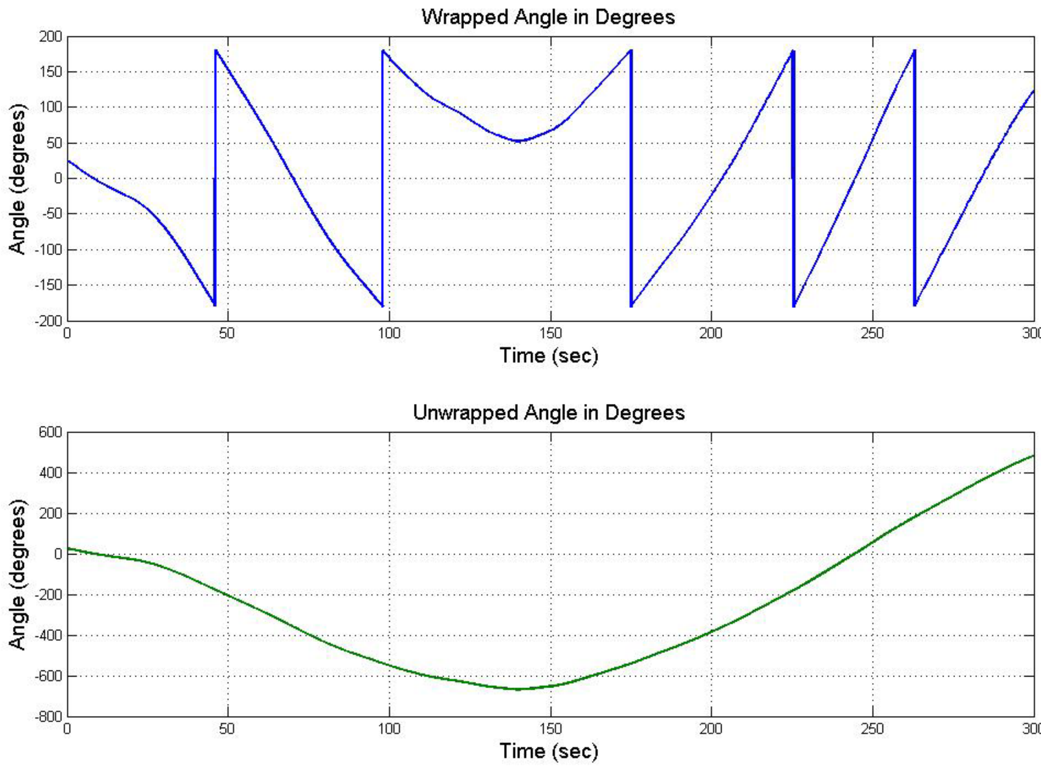

Figure 9. Wrapped and Unwrapped Angles

Figure 9 shows the reverse unwrapping operation of the wrapped angles that was shown in Figure 8. Quick comparison of the unwrapped angles in Figures 8 and 8 show that the two unwrapped angles are equal to each other. Rather, there is a fixed offset because the unwrapping operator does not know the "true" value of the unwrapped angle at time t_0. In some sense, there is no "true" value for any phase angle and all the angle discussions are based on a specific starting time used in place of the angle reference as noted in the previous sections.

Phase Angle Difference Calculation Methodologies

This section explains practical methods for calculating phase angle differences, and recommendations and considerations for each method. Appendix A provides real implementations of phase angle difference calculations from electric utilities with extensive synchrophasor networks and applications.

Simple Arithmetic Subtraction

Simple arithmetic subtraction of two phase angles is the simplest form of phase angle difference calculation. This technique works only if the time period of calculation does not include any wrapping of one of the phase angles around +/- 180 degrees. In this case, the raw arithmetic difference will give a false value unless wrapping is accounted for. This technique should be used with caution for quick calculations but avoided for any applications or tools using phase angle differences. Figure 10 shows the situation where the principal value from PMU A rotates from positive to negative around 180 degrees while PMU B principal value does not. Simple arithmetic subtraction results in a very large phase angle difference calculation, which is not physically accurate.

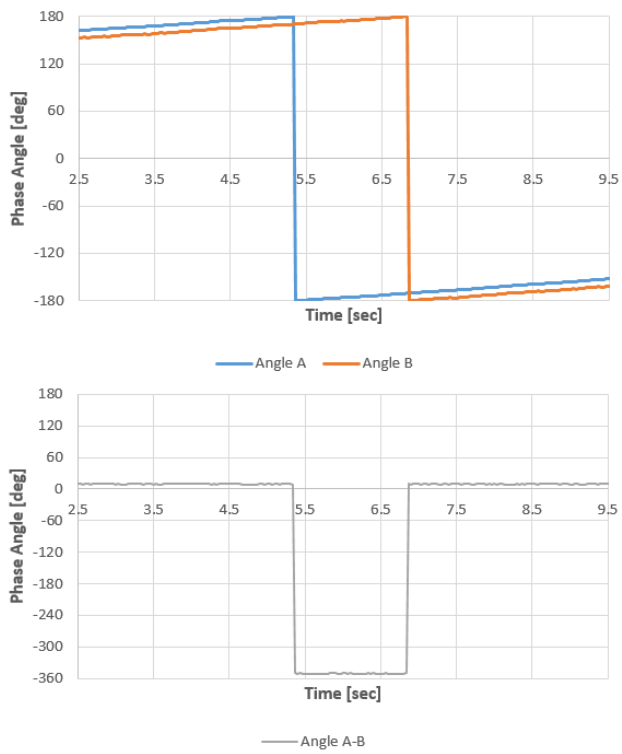

Figure 10. Arithmetic Subtraction Calculation Issue

Recommendations & Considerations: This method for calculating phase angle differences should be avoided, as it introduces calculation errors for any time series that includes a phase angle wrapping around +180 or -180 degrees.

Unwrap Then Subtract

The principal value of phase angle reported from the PMU can be unwrapped starting at some point in time, *t*, whether performing online or offline angle difference calculations. The point at which unwrapping starts is ambiguous from the calculation standpoint. When two phase angles are unwrapped, the angle difference can be directly calculated with a simple arithmetic subtraction because the discontinuities in phase angle around ± 180° are removed. Rather, the phase angle measurements are converted into a continuous measurement by detecting for these discontinuities and adding + or (-) 360° to an aggregate phase angle and the principal value reported by the PMU.

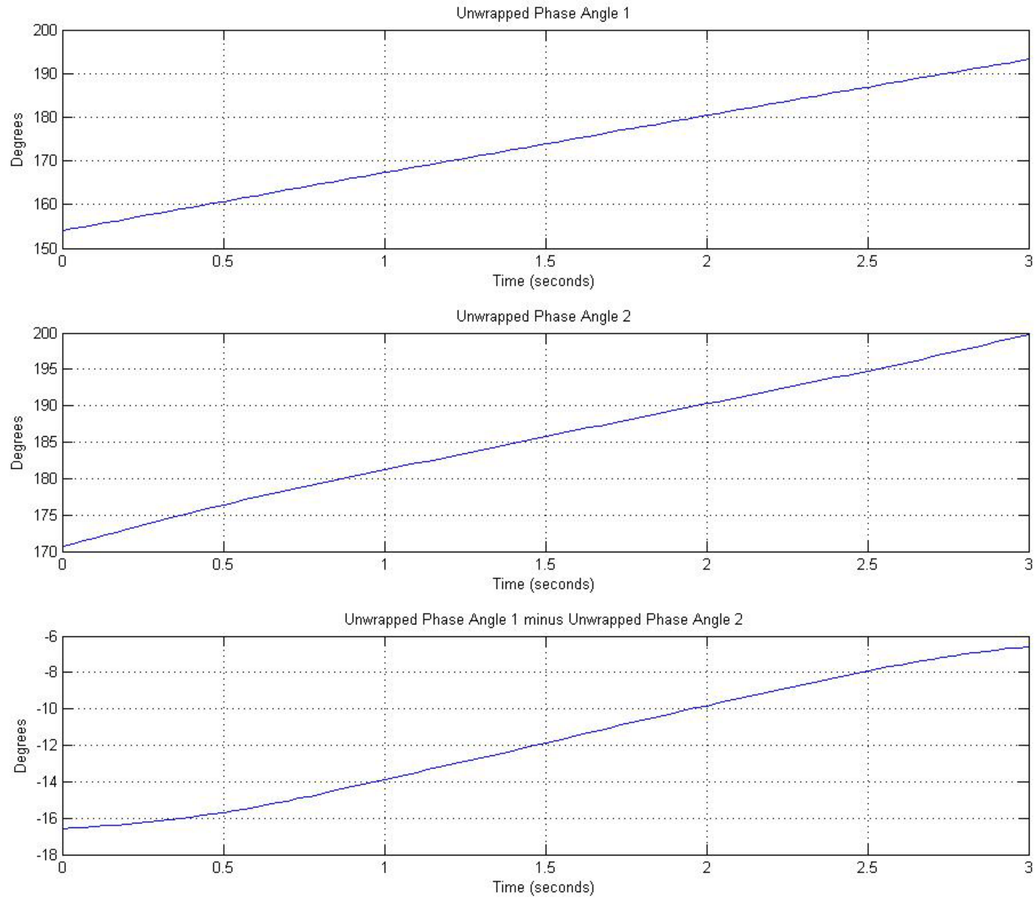

Figure 11. Phase Angle Difference Calculation with Unwrapped Angles (same data as Figure 6)

Recommendations & Considerations: This method for calculating phase angle differences is numerically accurate compared with vector mathematics. It can be difficult to implement in real-time EMS applications due to the complexity of the logic; however, it is very useful for offline applications.

Subtract Then Wrap

There are two methodologies for the 'Subtract Then Wrap' algorithm, which takes an arithmetic subtraction of two phase angles and then determines if any rotation or modulo effect must be applied to the calculation based on the result.

Phase Angle Rotation

One solution is to rotate both angles the same amount and direction so that one angle is 0.0 degrees. The angle difference is therefore the angle of the second quantity with appropriate sign adjustment. This calculation works with any two phase angles under the assumption that the difference is always less than 180 degrees. This is always true for voltage and current measured at the same electrical location; however, power system angles between buses could be more than 180 degrees for very large interconnected power systems. This is particularly true during transient conditions involving large power swings. This situation is a special case and requires special handling, but is something to be aware of for such systems. This has not been considered an issue thus far by grid operators in North American grids where angles can exceed 90 degrees between two points, but have not been observed to exceed 180 degrees.

Modulo Logic

Another approach is simply to subtract the angles in the direction needed for the particular calculation. If the difference is < -180, then add 360 to the difference. If > 180, then subtract 360 from the difference. This does the same thing as rotating both and accounting for the rollover.

```
double UnwrapAngleDelta(double AngleDelta)
{
    double rval = AngleDelta;
    if (Math.Abs(AngleDelta) > 180)
    {
        if (Math.Sign(AngleDelta) < 0)     //
is AngleDelta negative?
        { rval = AngleDelta + 360; }
        else // AngleDelta is positive
        { rval = AngleDelta - 360; }
    }
    return (rval);
}
```

Figure 12. Unwrap Then Subtract Logic

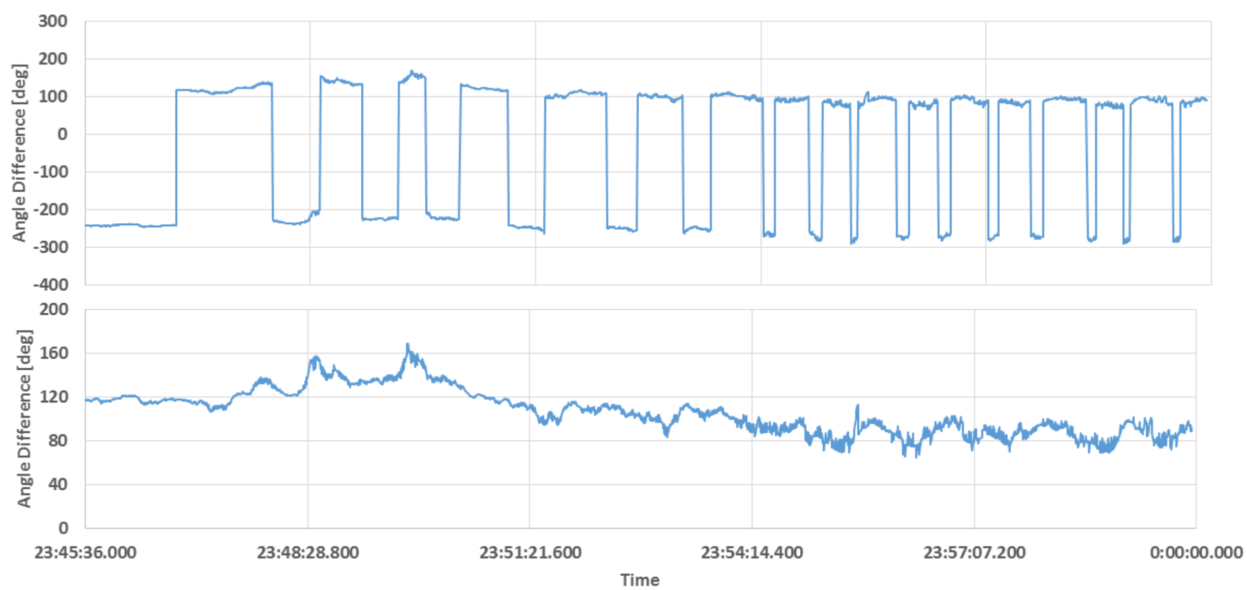

Figure 13. Raw Subtraction (Top) and 'Unwrap Then Subtract' Result (Bottom)

Recommendations & Considerations: This method for calculating phase angle differences is numerically accurate compared with vector mathematics. The code for implementing this logic in real-time and offline applications is fairly simple, making it a convenient and efficient algorithm to use.

Vector Mathematics

For offline applications, phase angle differences can be computed using two phasor vectors with a generic magnitude of 1.0 pu. The phase angle difference can be calculated as the angle of the multiplication of one phasor vector with the complex conjugate of the other phasor vector.

$$\alpha_{Diff} = \angle\left(\bar{V}_1 \cdot \bar{V}_1^{\,*}\right)$$

In engineering analysis programs, such as MATLAB, this is a convenient method for calculated angle difference between two synchrophasor phase angles for large blocks of data where computation speed and memory are not an issue.

```
V1 = 1+0i;
V2 = 1+1i;
dDiff = angle(V1*conj(V2))*180/pi   % in degrees
```

The vector math calculations can be used as a reference methodology.

Recommendations & Considerations: This form of phase angle difference calculation applies best to offline engineering applications. It is a simple means of calculating phase angle differences for programs that can easily handle complex numbers and complex conjugate computation. However, real-time tools and EMS applications do not have this computational ability nor can they handle the additional computation requirements in terms of memory or speed.

Phase Angle Difference Calculation Locations

The concept of phase angle differencing involves some form of calculation on two or more phase angle measurements. The PMUs report raw phase angle values calculated for an instant in time. The PDC fundamentally performs aggregation and time alignment functions. Data historians effectively store analog and digital values for retrieval and usage at some point in time. It is the applications that actually perform phase angle difference calculations on the PMU measurements. These applications, however, can be embedded in various locations in the PMU stream based on the purpose of using the phase angle difference. For example, the PDC may have advanced analytical functions built into it. Similarly, the data historian can likely perform mathematical operations on the data as well. However, these are forms of applications working directly on the streaming or stored phasor data. Applications are where the functionality and calculation actually occurs, whether integrated somewhere in the data stream or at the end of the stream where data is actually applied to some function or purpose (i.e., visualization, oscillation detection, etc.)

Regardless of the location in which the calculations occur, one must ensure the integrity of the phasor measurements being used for difference calculations. For example, any user of data that has been compressed or filtered must be aware of this for developing downstream application using the data. For example, Reliability Coordinators who receive phasor data from PMUs from different manufacturers will often not store any streamed phase angle differences calculations; rather, they will use or extract the raw phase angle values reported from the PMU and calculate phase angle differences themselves. On the other hand, a Transmission Owner that directly owns the PMUs and uses one manufacturer and understands all the modifications to the data may directly store difference calculations and use them in advanced applications. Regardless, there are downstream consequences that must be accounted for when using pre-calculated, filtered, or compressed forms of data including phase angle values.

Phase Angle Difference Calculation Considerations

Each method for calculating phase angle difference has benefits and drawbacks that should be considered when using them for either online calculations or offline analysis, including the following considerations:

- **Energy Management System (EMS) Calculation Capabilities:** Real-time EMS systems have limited computational capabilities. Performing some phase angle calculations may be limited by these capabilities, and some utilities are performing calculations outside the EMS for this reason.
- **Calculation Speed:** In real-time applications such as power system control, calculation speed is critical and must be minimized to the extent possible. Some calculations require significant computation power, and hence time, while others are simple numerical operations.
- **Calculation Accuracy:** Calculation accuracy is essential for all applications using PMU data and phase angle differences. Some calculation methods have higher levels of accuracy than others for different conditions and assumptions.
- **CPU Usage:** Designing a robust synchrophasor network and application platform requires that sufficient CPU power is available to perform calculations. It is important to consider the CPU usage of different applications and calculation methods to design an effective architecture.
- **Data Storage:** Data storage is critical and costly, particularly for any high resolution data source such as PMU data. Angle difference calculation methods have different storage requirements.
- **Missing Data:** Intermittent or continuous dropped data can have a significant impact on some phase angle difference calculation methods while having minimal or no impact on other calculation methods. Missing or bad data must be accounted for when selecting a phase angle difference calculation method since some methods must account for wrapping while other do not.
- **Angle Resetting:** Angle resetting may be required for calculating accumulated phase and therefore must be accounted for in some methods; however, other methods cannot determine accumulated phase and therefore do not need to consider angle resetting.
- **Application Requirements:** Simple arithmetic subtraction of phase angles is not suitable for real-time or offline analysis, except for short time window calculations where neither angle measurement used in the difference calculation crosses ± 180°.
- **Absolute Time Error Correction:** "Unwrap Then Subtract" can provide operators with the absolute time error correction by maintaining a calculation of the accumulated phase angles; other methods do not have this capability.

Table I. Phase Angle Difference Calculation Consideration

	Simple Arithmetic Subtraction	Unwrap Then Subtract	Subtract Then Wrap	Vector Mathematics
EMS Calculation Capabilities	Achievable	EMS computation limitations	Achievable	Not possible
Calculation Accuracy	Poor, when raw angles wrap	Accurate calculation. Same level of accuracy.		
Calculation Speed	Fast			Slow
CPU Usage	Minimal			
Data Storage	Minimal	High	Moderate	Minimal
Missing Data Issues	Minimal issues; only calculations with missing data voided	Missing data can result in wrap around points getting missed	Minimal issues; only calculations with missing data voided	Minimal issues; only calculations with missing data voided
Angle Resetting	None	User can specify rate of resetting[4]	None	None
Application Requirements	Not acceptable	Acceptable		
Absolute Time Error Correction	Not achievable	Required	Not achievable	Not achievable

Accumulated Phase Angle Difference

The relative "phase" of two signals that are close in frequency will change slowly with time.[5] If the frequencies are both constant, the phase will change as a linear function of time. For example, suppose there are two signals: one frequency is 60 Hz exactly, and the other is 60.1 Hz. At t = 0, let us say that the relative phase is zero. After one second, the higher frequency signal has advanced a tenth of a cycle, or 36 degrees. After ten seconds, the signal may appear to be in phase, but in fact one signal is 360 degrees ahead of the other. After 4000 seconds (just over an hour), the phase advance is 400 cycles, or 14,400 degrees.

But at the same time, the principal value is zero.[6] The principal value is what the PMU measures. The PMU is not designed to keep track of the evolution of relative phase of two signals since some arbitrary time zero, it is designed to measure and report the principal value.

[4] Angle resetting helps mitigate phase angle error accumulation using the Unwrap the Subtract method.
[5] "Phase" is in quote marks because the relative phase of two signals that are not at the same frequency is not a defined entity. In the usual sense of the word phase, the relative phase would be a function of the time. We shall see later that there is a way to solve the problem.
[6] Strictly, the values defined on the interval $(-\pi, \pi]$ are the principal values. The notation indicates that $-\pi$ is included, but not $+\pi$. That keeps the principal values single-valued.

The reason for that is simple: the PMU sees the input signals for a very short time. Given only the short amount of time of the PMU measurement window, the system has no choice but to report the principal value. To consider again the example above, unless the PMU stored phase information for ten seconds, it could not know that the phase was not zero degrees but 360°. And one hour of storage of the phase values would be needed to give the true relative phase of 14,400° after an hour, and so on.

So that is what accumulated phase is about. The PMU has no knowledge of it. It must be obtained from PMU signals by "unwrapping" the measurement results as they evolve in time.

Phase Angle Distance

Phase angle distance is defined as the number of degrees of separation between the GPS reference cosine wave and the point of measurement in the grid. The wrapping angle reported from the PMU is the difference between the measured angle and the reference angle (always zero). Consider a bus with a frequency of 60.02 Hz. The phase angle of the measured bus is changing at 7.2 degrees per second since frequency is defined as the rate of change of phase angle. For example with the frequency at 60.02 Hz, starting from midnight UTC the bus angle is 360 degrees ahead of, or 16.667 ms faster than, the GPS reference bus as defined by the Time Error by NERC[7]. Suppose the bus frequency remains at 60.02 Hz for one hour. The "angle distance" is now 25,920 degrees "ahead" of the reference, or 1.2 seconds faster than UTC. However, the principal value reported by the PMU is zero degrees (0 = MOD(25,920,360)), but there is a difference of 72 cycles between the reference and the measured bus. The number 72 is called the ROC (rollover counter) as described in more detail below.

Figure 14 shows a sample plot of unwrapped phase angles over a 3.04 hour period. The unwrapped phase angles are smooth until midnight UTC time (PDT + 7h), where the phase angle distance is artificially returned to 0 degrees (accumulated phase angle distance can be returned to zero at any point in time with any time resolution. The angles plotted range from -30,000 to +5000 degrees. This shows that the grid was fast in the early afternoon and then became slower later in the day, likely due to additional load late in the afternoon. By 5 pm local time, the grid was about 1.38 seconds slow (30,000/(360*60)). In some respect, this provides operators with information regarding accumulated offset and operators can make adjustments to generation-load balance to return the system near a nominal frequency with low accumulated phase angle.

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[7] Balancing and Frequency Control, A technical document prepared by the NERC Resources Subcommittee, January 26, 2011. p. 13.

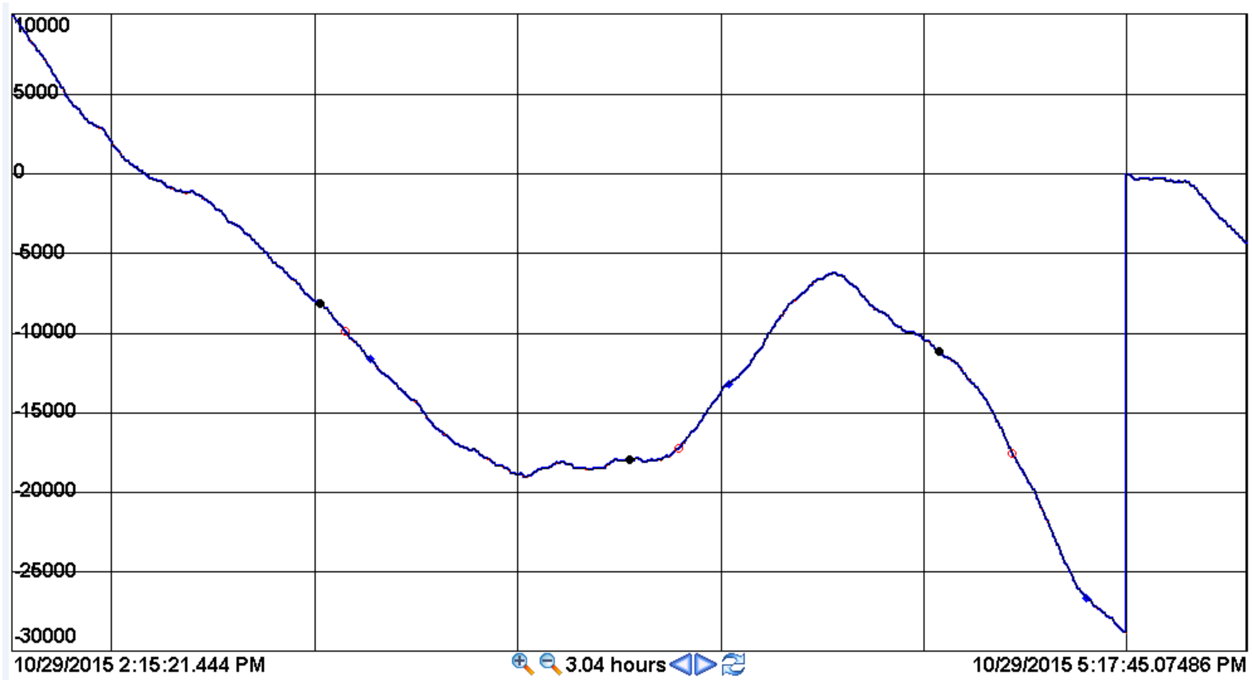

Figure 14. Unwrapped angles and arbitrary reset

The angle differences for the three unwrapped angles from Figure 14 are shown in Figure 15, with one angle used as a reference signal for the two plotted angle differences. The time range here is four hours and the angle scale for the phase angle differences is between 2.4 and 6.4 degrees. Using the unwrapped angles, and even with the discontinuity to return the accumulated angle distance to 0 at midnight UTC, the angle differences do not have any significant discontinuities.

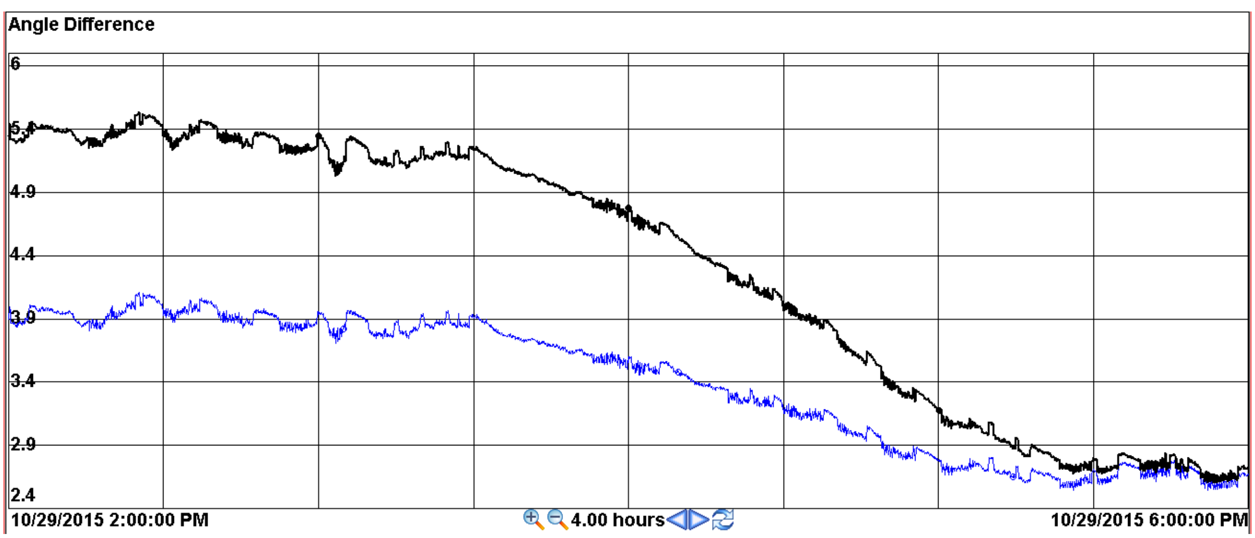

Figure 15. Angle differences using unwrapped angles

Rollover Counter (ROC)

The Rollover Counter (ROC)[8] for a phase angle measurement say $\phi(t_k)$ is defined as the count of total rollovers or wrap-arounds in the phase angle since a specified starting time, say t_0. When the phase angle rolls over from +180 degrees to -180 degrees, ROC is incremented by 1. Conversely, when the phase angle wraps around from -180 degrees to +180 degrees, ROC is decreased by 1.

As discussed in the previous sections, the PMU phase angle is measured with respect to Universal Time Clock as the phase reference. Therefore, every positive wrap-around or positive rollover from 180 degrees to -180 degrees implies that the phase angle $\phi(t_k)$ has gained an additional one cycle or an extra 1/60 seconds in time error with respect to the UTC time reference. In other words, ROC counter $ROC(t_k)$ for any phase angle $\phi(t_k)$ quantifies how many cycles the time error differs with respect to UTC at time t_k since time t_0. Therefore, the units of $ROC(t_k)$ can be thought of as multiples of 360 degrees in terms of change in accumulated phase measurement since time t_0, or as multiples of cycles in the context of accumulated time error with respect to UTC time since time t_0.

The unwrapped phase angle $\Psi\big(\phi(t_k)\big)$ at time t_k can be shown to be:

$$\Psi\big(\phi(t_k)\big) = \phi(t_k) + 360\ ROC(t_k)$$

Figure 16 shows an example of ROC calculation for a phase angle measurement shown in the top subplot[9]. There are three rollovers from +180 degrees to -180 degrees which imply that the ROC counter (middle subplot) changes from 0 to 3 over this time period. The bottom subplot shows the corresponding unwrapped phase angle which is growing well over 180 degrees.

---

[8] V. Venkatasubramanian, "Real-time strategies for unwrapping of synchrophasor phase angles", *IEEE Trans. Power Systems*, to appear.

[9] The "gaps" in the phase angle plot seen in Figure 16 correspond to time periods when the PMU was not reporting or when the data was reported as NaN (Not-A-Number) by the PDC.

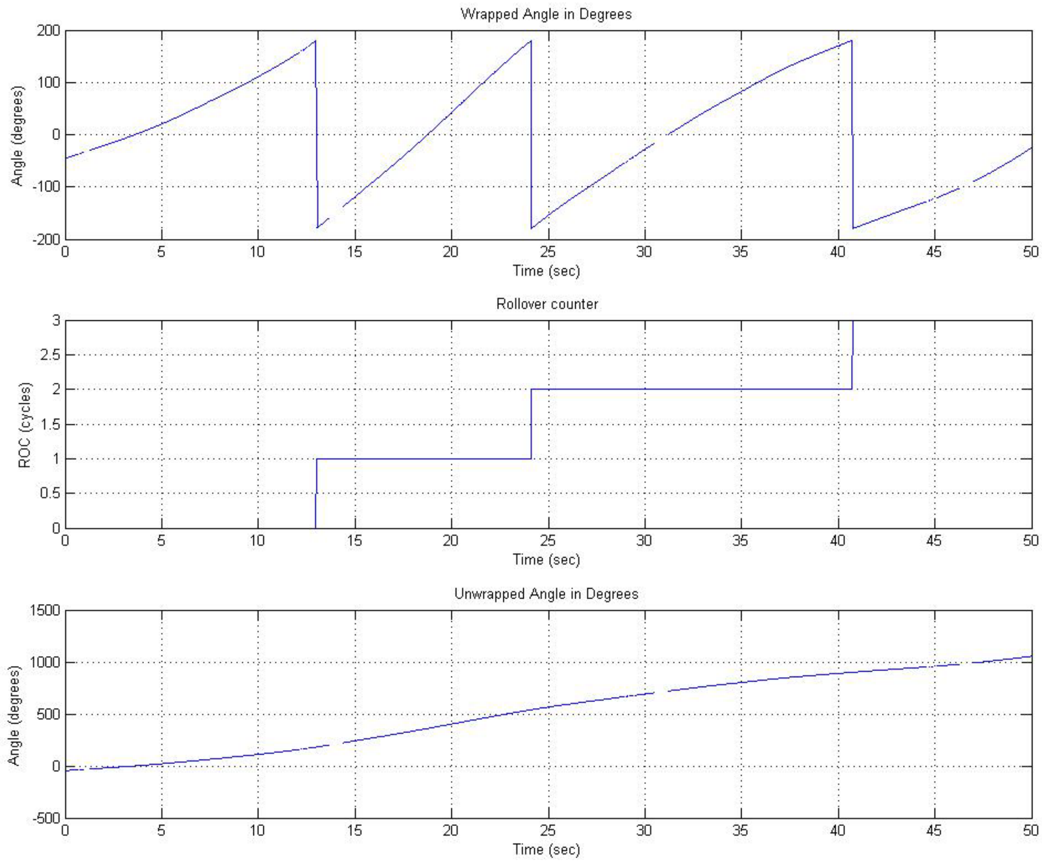

Figure 16. Rollover Counter (ROC) Example

As the ROC is a multiplier to the principal value, it is easily accounted for in applications to attain the absolute angle since the last reset. This makes it computationally and qualitatively easier to track both the PV and the absolute angle for different applications' purposes.

Time Error

Before the advent of quartz time mechanisms, most of the wall clocks kept track of time by counting the number of cycles in the AC voltage signal measured in wall outlets. A count of 60 cycles could be directly interpreted as a tick of one second. When the AC bus frequency fluctuates above and below 60 Hz, the wall clock will then gain or lose time respectively in such mechanisms. In this context, NERC has had a policy on keeping the time error close to zero over the long term. This policy is still in effect even though most of the wall clocks have moved on to quartz timing mechanisms over the past decade. The time error between the time seen by any such wall clock running off a local AC bus frequency and the universal time reference since the time the grid was synchronized is known as the absolute time error. In the context of ROC counters introduced above, the absolute time error can be precisely quantified as the product of ROC count times 1/60 seconds, assuming the ROC count was initialized when the grid was synchronized.

If the ROC count $ROC(t_k)$ at time t_k of any bus voltage phase angle $\phi(t_k)$ was initialized at some arbitrary time t_0, the relative time error seen by the wall clock running off of the bus frequency $\omega(t_k)$ (which is the time derivative of the bus voltage phase angle $\phi(t_k)$) will be the product of

ROC(t_k) and 1/60 seconds. Whereas the absolute time error cannot be measured for a large power grid such as the eastern interconnection (because of lack of continuous time, frequency or phase angle measurements from the time the grid was synchronized), the relative time error starting from a specified time instant t_0 is easier to measure and to keep track of from a practical viewpoint. This is especially true in the context of synchrophasors using the ROC counters defined above because the synchrophasor phase angle is directly measured by using the UTC time as the reference signal for any synchrophasor phase angle measurement as discussed in the earlier sections.

Concluding Remarks

This paper discusses various aspects of using phase angle measurements for synchrophasor applications. The principal value reported by PMUs is resolved to the interval (+180,-180], which causes complexity in difference calculations and other applications that must be dealt with appropriately. Key points of conclusion include these:

- Robust synchrophasor applications must account for principal value of the phase angles reported by PMUs resolving to the interval (+180,-180] in any difference calculations.
- The problem statement related to phase angle differences and definitions of angle wrapping and unwrapping are clearly defined in this report for industry reference.
- Commonly used phase angle difference calculation methods are provided for references; each method has pros and cons that must be accounted for in any application.
    a. Vector mathematics is numerically accurate but computationally complex, particularly for real-time applications. This method is often used to benchmark other methods to ensure their accuracy and is recommended for offline engineering applications where available.
    b. The “Subtract then Wrap” and “Unwrap then Subtract” methods each have their benefits and drawbacks in terms of implementation; however, both are considered accurate methods for performing phase angle difference calculations.
    c. Simple Arithmetic Difference calculations should be avoided for both real-time and offline applications; any PMU phase angles that resolve around (+180, -180] will cause numerical issues with this method, resulting in incorrect results.
- There are a number of considerations that affect the choice of which difference calculation method to use for a specific application, including calculation capabilities, accuracy, speed, CPU usage and data storage, and ability to handle missing data.
- Accumulated phase is discussed in this paper, which uses unwrapped angle measurements for any phase angle calculations. Two similar yet different approaches include Phase Angle Distance and Rollover Counter (ROC). Both track unwrapped angles over a given time to determine the accumulated phase since some time t_0.
- Due to inconsistency with frequency measurements across different PMU manufacturers and configurations, frequency-based synchrophasor applications may opt to calculate frequency from the reported phase angles directly.
- Utilities are using phase angle measurements for different applications across North America and using different methods for performing angle difference calculations (Appendix A).

Appendix A: Actual Implementations of Angle Difference Calculations

PJM Implementation

PJM initially used phase angles with voltage and current magnitude as part of a first-round data value reasonability check, in particular when integrating new PMU data. For example, a simple and fast check would be to compare SCADA-sourced MW telemetry against synchrophasor system-derived MW telemetry:

$$MW = 3\, V_+\, I_+\, cos(\delta_V - \delta_I)$$

Presently, more advanced data accuracy checking is occurring within the PJM Linear State Estimator (LSE), which consumes phase angles from the synchrophasor system and breaker statuses from the EMS. The LSE automatically detects and reports on deviant phasor values as one of the operational values it provides from full state solution.

Phase angles were experimentally added to certain EMS displays of critical EHV tie lines. This was found to be of only moderate value for dispatch-oriented displays, but still includes voltage angle and current angle at each end of a line and calculated angle difference across the line. Earlier, an alternate equation was used that has good results:

$$IF\ (\theta_a > \theta_b) AND (\theta_a < \ \theta_b + 180) OR\ (\theta_a < \theta_b - 180)$$

$$THEN\ \theta_{a-b} = -\,acos(cos(\theta_b - \theta_a))$$

$$ELSE\ \theta_{a-b} = acos(cos(\theta_b - \theta_a))$$

There is a significant computational burden to perform this early method compared to the much simpler "Subtract Then Wrap" method described elsewhere in this document and so has very little actual practical utility. However as a minor note, the "Subtract Then Wrap" method (where 360° is added or subtracted from the difference) generally requires an IF THEN, ELSE IF, ELSE or otherwise an initial subtraction operation before an IF THEN, ELSE statement. If by chance this is not possible in the host system or environment because of inherent limitations, perhaps the above alternate calculation above requiring only an IF THEN, ELSE statement might suffice.

Otherwise, PJM has implemented the more common "Subtract Then Wrap" method. For example, in the OSIsoft PI historian this is implemented as a PI Asset Framework formula.

A=Positive Sequence Angle;

B=Positive Sequence Angle;

[if(B - A > 180) then B - A - 360 else if(B - A < -180) then B - A + 360 else B - A]

The results are not stored as separate values (PI tag) but are available for real-time calculation on demand on either real-time or historical data. Obviously this results in storage savings at the moderate cost of some real-time calculation load.

More directly, PJM's synchrophasor vendor products handle these calculations internally, and often those results are incorporated tightly into the end applications such as the polar chart shown in Figure 17.

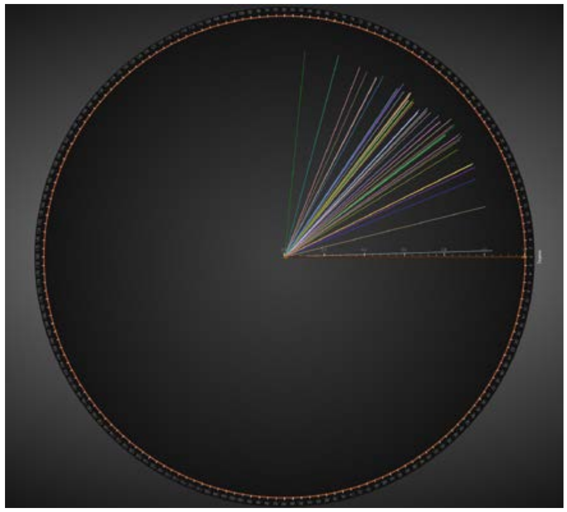

Figure 17. Phase Angle Polar Chart

The polar chart is an example of multiple phase angles being compared (subtracted) from the same reference[10] phase angle. The reference phase angle should come from a PMU that is of proven reliability and accuracy since it will affect all of the displayed values. Applications that use a reference PMU should have the ability to fail to an alternate reference PMU when failure of the primary reference PMU is detected.

Other advanced applications currently in production use, such as oscillation detection, are being further developed in tight integration with vendor technology and so low level management or individual calculation manipulation of phase angle differences is not often necessary.

[10] This use of the word "reference" refers to selecting a reference phase angle measurement to be used in all associated angle difference computations such that all angles are determined in a consistent fashion relative to each other.

Bonneville Power Administration (BPA) Implementation

BPA uses angle differences in a variety of ways for online and offline tools. Online calculation methods implemented by BPA are discussed here. BPA uses angle differences for the following applications:

- **Voltage Angle Differences** – Phase angle differences are derived from PMU bus voltage phasors and used for a variety of purposes including:
  - Path Alarming – Alarms are triggered when individual segments on a path exceed defined thresholds.
  - Wide Area Angles – Phase angle differences of interest are displayed visually for dispatchers and real-time engineer awareness.
  - Implied Power Flows – Phase angle differences are used to visually show implied power flow directionality along major transmission lines. This display interacts with the Frequency event Detection Module (FDM) and visually shows the implied changes in power flows immediately following a frequency event.
- **Sample to Sample angle differences** – Phase angle differences between two samples from the same voltage phasor are calculated and used for the following:
  - PMU Frequency Calculation – Since frequencies reported by PMUs can vary relatively significantly between PMU manufacturers while phase angle calculations are much more standardized, BPA calculates frequency from reported phase angles using the following formula:

    $$Frequency = 60 + [Angle_n - Angle_{n-1}] * SampleRate/360$$

  - Synchrophasor RAS Algorithms
  - Oscillation Detection Application
  - Mode Meter Application

BPS's online tools use the "Subtract Then Wrap" method. (Note - this does not apply to the Modal Analysis Software libraries). BPA uses the following algorithm:

```
deltaAngle = angleA - angle B
if (deltaAngle > 180.0)
        deltaAngle = deltaAngle - 360.0
else if (deltaAngle < 180.0)
        deltaAngle = deltaAngle + 360.0
```

When implementing angle differences, vector subtraction was considered to be the gold standard but computationally intensive, so extensive testing was done to ensure that the above formula was equivalent to vector subtraction.

BPA also implements an islanding detection algorithm, but it uses frequencies rather than phase angle differences. While frequencies can be calculated from the phase angle changes, BPA currently uses the PMU-calculated frequency.

Peak Implementation

Peak's phase angle calculation method is still under investigation at this time. Peak RC is planning to create Phase Angle Delta values calculated using down sampled voltage phasors on approximately 26 identified Phase Angle Pairs. Two methods are being considered:

1. All down-sampled (1 sample per second) phasor values are presently being sent from the openPDC into the Alstom[11] eTerraBrowser. Calculate the phase angle delta using the built in eterraBrowser calculation engine.
2. Send the down-sampled (1 sample per second) phasor values directly to OSI soft PI then use the PI analytics engine to calculate the phase angle delta.

In either case, the simpler "Subtract then Wrap" methodology will be used. The calculated phase angle differences will be made available to all Peak RC members via ICCP.

---

[11] Now General Electric

American Transmission Company (ATC) Implementation

ATC scans voltage magnitude and angle data from all three phases for all PMUs and stores it at full resolution in an OSIsoft PI database. No compression or exception processing is used on the angle data, as the system does not work well with the transitions from +180° to -180°. ATC does not store positive sequence angle data provided by the PMUs in the PI historian to reduce tag counts and storage space requirements, since this data can be calculated from the phase data if necessary.

ATC does not currently use PMUs for state estimation; however, a PERL script has been developed that retrieves the phase A voltage angle for each PMU from the historian. The program uses a list of preferred reference stations based on the historical availability and data quality metrics to ensure higher likelihood of a valid reference angle selection. The application chooses one of these signals that is available and uses that as the angle reference. It then calculates the relative angle differences for all other buses based on that reference bus. If there are issues with PMU-reported angles crossing +/-180 that impact the results, the following code is used to adjust those angles:

```
if ($ReferenceAngleFound eq "True")
{
        $Value = $ValueHash{$TagName} - $ReferenceAngle;
        if ($Value > 180)
        {
                $Value = $Value - 360;
        }
        elsif ($Value < -180)
        {
                $Value = $Value + 360;

        }
        $Quality = $GoodQualityValue;
}
else
{
        $Value = $ValueHash{$TagName};
        $Quality = $QuestionableQualityValue;
}
```

The PERL script runs every 20 seconds and creates a file with the data. The ICCP FileLink application transfers the calculated angle data from the PMU data historian to the Energy Management System (EMS). EMS has the ability to write the solved voltage angle from each State Estimator (SE) solution to an analog value. That data is compared to the real-time PMU angles supplied by the SE program. That data also uses a reference bus and calculates angle difference from that bus for the analysis.

Test tools are under development for benchmarking SE results with data reported by the PMUs as shown in Figure 18. This data will help identify modeling issues (or PMU data quality issues) that were not visible using only SCADA data. This is still in the development state, but part of ATC's long range plan.

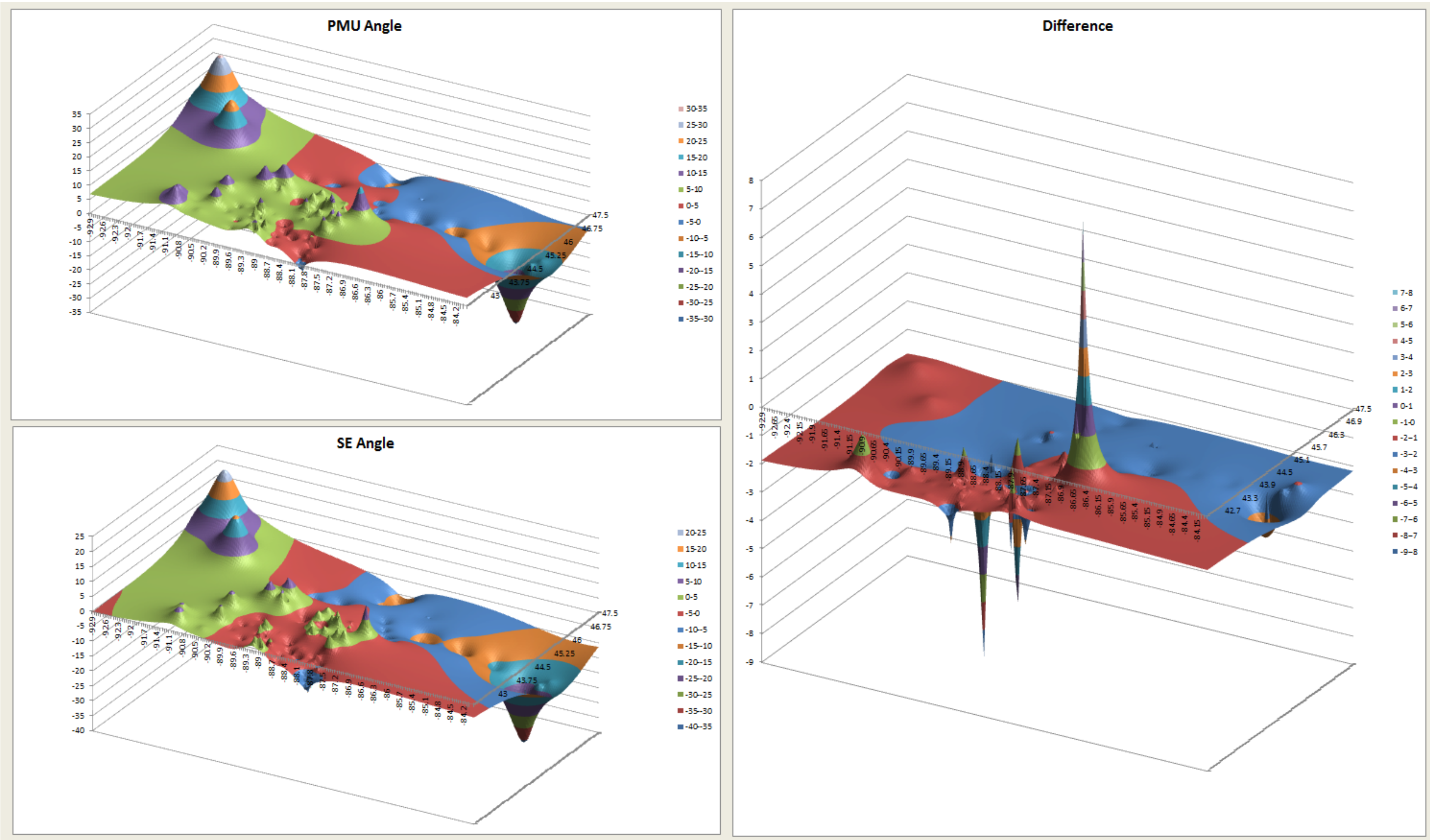

Figure 18. 3-D State Estimator Benchmarking Tools for ATC

Dominion Virginia Power (DVP) Implementation

Dominion Virginia Power (DVP) uses phase angle unwrapping only for PMU commissioning. DVP found that it is much easier to check PMU angles for installation correctness and accuracy if they are unwrapped. Things considered include:

- No large angle jumps or discontinuities
- No significant missing data
- No significant repeated values
- No significant offsets or other errors

When the phase angles are wrapped (raw from the PMU), all the phase angles out of one PMU can end up looking like what is shown in Figure 19. This makes it very difficult to check each angle individually.

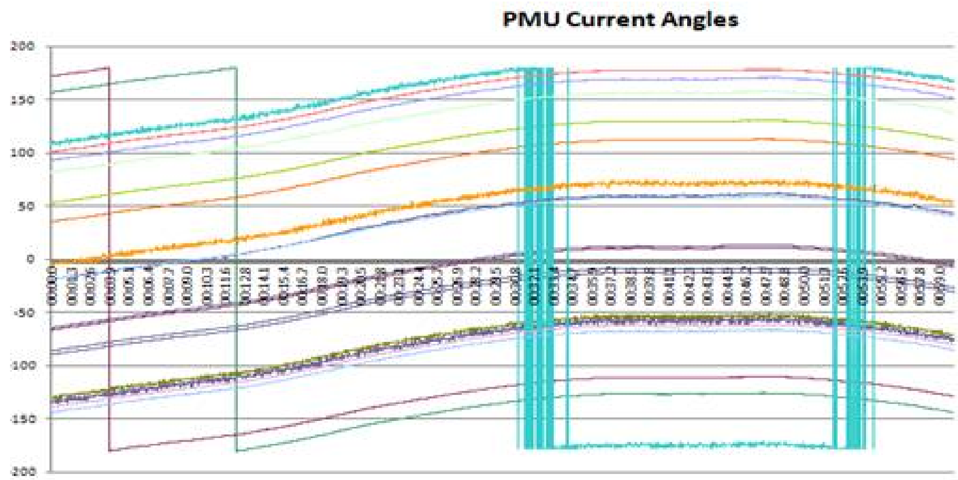

Figure 19. Wrapped Angle – Difficult to Discern Errors for Commissioning

DVP uses the following logic for phase angle unwrapping:

```
' Angle Unwrapping
If (Angle value - Past angle value > 350), Then
        multiplier = -1
If (Angle value - Past angle value < -350), Then
        multiplier = 1
Unwrapped angle value = Angle value + (multiplier*360)
***350 value can be altered based on possible angle jumps.  350 allows for worst case
angle jump of 10 degrees (ex: angle=179, then jumps 10 degrees to -171, with phase
angle wrapping).

Examples:
-----------------
New angle = -179.999
Old angle = 179.999
Diff = New angle - Old angle = -359.998
Need unwrapped angle to be = 180.001
New unwrapped angle = -179.999 + (multiplier*360) = 180.001
Multiplier must be = 1
```

New angle = 179.999
Old angle = -179.999
Diff = New angle - Old angle = 359.998
Need unwrapped angle to be = -180.001
New unwrapped angle = 179.999 + (multiplier*360) = -180.001
Multiplier must be = -1

OSIsoft Implementation

OSIsoft PI system uses unwrapped angles, applying the following pseudo-code for angle distance unwrapping

```
/* PSEUDOCODE
/* note: 'previous_unwrapped' and 'rawDiff' are DOUBLE PRECISION floats
/* note: user must update ‘previous_unwrapped’ with the return value, prior to subsequent
calls
IF reset_time
        Return angle_now
ELSE
        rawDiff = angle_now - previous_unwrapped
        Diff = MOD ( rawDiff, 360 )
        IF Diff > 180 Diff = Diff - 360
        ELSE
        IF Diff <= -180 Diff = Diff + 360
        ENDIF
ENDIF
Return previous_unwrapped + Diff
/*
```

This algorithm only needs to store one variable: “previous_unwrapped”. The difference between the previous unwrapped angle and the current angle measurement is accumulated while taking into account the discontinuity when the raw PMU angle wraps. If needed, the wrapped angle can be recovered by dividing the unwrapped angle by 360.

Washington State University (WSU) Implementation

The algorithm continuously maintains a rollover counter ROC_i for every phase angle δ_i. ROC_i is incremented by +1 or -1 depending on whether there is a roll-over in δ_i from +180 to -180, or -180 to +180, respectively. Then, at any time, the corresponding unwrapped phase angle unwrapped_δ_I can be calculated by adding 360 ROC_i with δ_i. the algorithm below can handle missing data or NaN values in the data streams and rare cases of double rollovers when a phase changes from +360 to -360 or -360 to +360 degrees, which can happen in phase angle difference calculations.

```
% Initialization
ROCi(t0) = 0;
LastKnownValuei = δi(t0);
For (tj > t0);
  If δi(tj) = NaN;
  % If value is NaN, do nothing.
  then
    ROCi(tj) = ROCi(tj-1);
  else
              Find N that minimizes
      |δi(tj) − LastKnownValuei + 360 N|.
    % N equals # of rollovers.
    % For transition from 180 to -180, N equals 1.
    % For transition from -180 to 180, N equals -1.
    % Increase the rollover counter by N.
    ROCi(tj) = ROCi(tj-1) + N;
    Unwrapped_δi(tj) = δi(tj) + 360 ROCi(tj);

              % Update the last known value.
              LastKnownValuei = δi(tj);
       end; % continue processing
end;
Unwrapped_δi(tj) = δi(tj) + 360 ROCi(tj);
```

Appendix B: Calculating Frequency from Measured Phase Angle

Frequency, f, of the phasor can be derived from the synchrophasor angle, δ°, reported by the PMU for a given reporting rate, Δt.

$$f = f_{nom} + \frac{\delta^{\circ}{}_{n} - \delta^{\circ}{}_{n-1}}{360^{\circ} \cdot \Delta t}$$

For example, a PMU reporting at 30 samples per second has a Δt value of 1/30 seconds. Phase angle differences are calculated between two consecutive samples, with adjustments made for wrapping around +180 degrees and -180 degrees. The result of the calculation is an offset from nominal 60 Hz frequency, therefore the result is added to the nominal (e.g., 60.0 Hz) frequency to get the actual frequency.

PMUs report frequency as part of the IEEE standard message. Frequency measurement performance requirements were introduced with the standard IEEE C37.118.1-2011; previous standards had no frequency measurement requirements. Consequently, frequency measurements from different manufacturers and vintages of PMUs may be inconsistent. In some cases, more consistent frequency estimates have been obtained by calculating frequency from PMU-reported phase angle differences. This is possible since the requirements on the synchrophasor estimate are rigorously defined. The new standard, C37.118.1-2011 with amendment C37.118.1a-2014[12], includes requirements for frequency measurement under both steady-state and dynamic conditions, so measurements from PMUs complying with this standard are expected to be consistent. Frequency measurements are generally required to have a steady-state accuracy of 5 mHz and be time-synchronized. The requirements vary with specific conditions; see the standard and amendment for details.

Figure 20 highlights some of the differences that are sometimes seen, and the result of calculation from phase. The figure shows how three different PMUs from different manufactures deployed at different utilities react to the same frequency event. In this figure, the dashed lines are PMU-reported frequencies while the solid lines are frequencies calculated from the phase angles. Since the PMUs are in electrically different parts of the system, it is expected that the reported frequencies will differ between PMUs. However, it is interesting to note (1) the frequencies calculated from phase differ from the PMU-reported frequencies, and (2) the variations in how the calculated frequencies differ from the reported frequencies.

[12] The amendment was written to better align synchrophasor, frequency, and ROCOF performance requirements for improved consistency and better implementability.

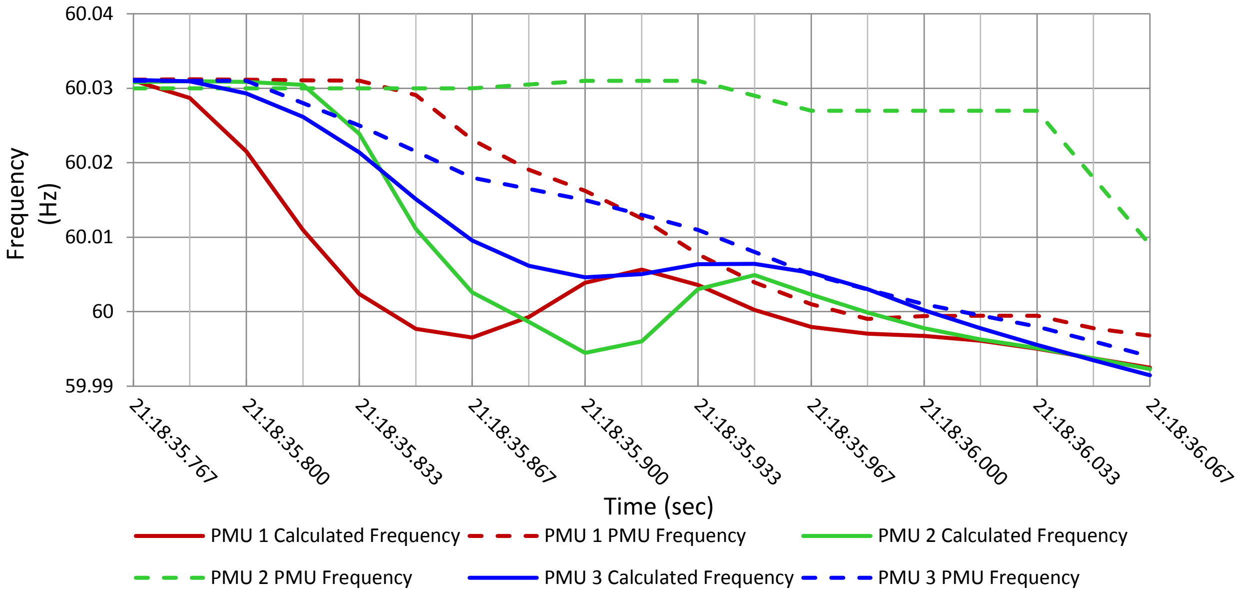

Figure 20. PMU Reported vs. Calculated Frequencies

The primary cause of the differences between the reported and measured frequencies is in how the PMU averages the frequency data. The delays between the solid and dashed lines from a given PMU are representative of the amount of averaging being done by the PMU.

Averaging the frequency data is not bad in itself; it is the inconsistent averaging that causes the most problems. Additionally, different applications may desire different amounts of averaging. Calculating the frequencies from the reported phase angles allows applications to apply the necessary amount of averaging in a consistent manner regardless of PMU manufacturer or PMU frequency configuration settings.

An example application where this is critical is event detection and location identification using frequency measurements. Large frequency excursions propagate through the grid with the closest PMUs responding first. In Figure 20, the reported frequencies imply that PMU 3 was closest, followed by PMU 1 and then PMU 2. The calculated frequencies imply that PMU 1 was closest followed by PMU 3 and then PMU 2, a very different order; but the fact that the averaging methods on the PMU-reported frequencies are not consistent between the PMUs leads to this inconsistent result.