

## **Guidelines for Siting Phasor Measurement Units**

North American SynchroPhasor Initiative  
Research Initiative Task Team (RITT) Report

June 15, 2011

### **List of Contributors:**

Joe H. Chow (RPI), editor  
Lisa Beard (Quanta-Technology)  
Mahendra Patel (PJM)  
Patrick Quinn (MISO)  
Alison Silverstein (NERC/NASPI)  
Dejan Sobajic (NYISO)  
Luigi Vanfretti (KTH, Sweden)

## 1. Introduction

Synchrophasor technology is relatively new to many utility companies as they plan for its introduction as part of their smart grid infrastructure investment. A first question that arises in planning for synchrophasor technology use is where should we place the phasor measurement units (PMUs)? Because synchrophasor data can be used in a variety of applications, the placement of PMUs should be driven by the intended applications, which may have varying data needs and differing PMU siting considerations. This white paper intends to provide information on and be a resource for such PMU placement issues, using recently developed PMU placement documents from various power grid operators and from applications described in technical papers.<sup>1</sup>

This white paper offers some guidelines for designers and users of the synchrophasor systems to decide where to place new PMUs. The paper addresses the information requirements of power system applications under all conditions (normal, alert, emergency, restorative) and across all time horizons (monitoring, protection, control, operation, planning).

As many PMU placement documents are already available, the main sections of this document will be quite brief. Some expanded details on the guidelines can be found in the appendices.

### 1.1 Dynamic measurement units

The common characteristic of dynamic measurement and recording units (DMUs) is their ability to sample and record voltages and currents in electric power systems at high sampling rates (presently defined as greater than or equal to 30 samples per second). Although many of the functions of these separate devices are converging with expanded electronic and storage capabilities, principal DMUs now in use include:

- Substation Fault and Event Recorders (time-stamped to GPS/UTC)
  - Digital Fault Recorders (DFR) and Sequence of Events Recorders (SOER) are local high-speed sampling devices that record transient and switching phenomena for specific substation equipment. SOER recording capability can also be provided by SCADA, RTU, generator Digital Control System (DCS), and DFRs.

---

<sup>1</sup> Recently developed PMU placement documents from various power grid operators include:

1. WECC – W. Mittelstadt, et al., “Disturbance/Performance Monitoring Plan Siting Requirements,” Disturbance Monitoring Work Group, June 29, 2001.
2. CERTS – C. Martinez, M. Parashar, J. Dyer, and J. Coroas, “Phasor Data Requirements for Real Time Wide-Area Monitoring, Control and Protection Applications,” White Paper for EIPP Real Time Task Team, January 26, 2005.
3. MISO – T. Bilke, J. Kleitsch, and T. Weekes, “Preferred PMU Placement,” April 3, 2009.
4. ATC – “PMU Placement Weighting Calculation”
5. WECC – D. Kosterev, et al., “WECC PMU Placement Criteria,” Rev. 3.0, February 2010.
6. NERC – NERC Standard PRC-002-1, “Regional Disturbance Monitoring and Reporting Requirements,” draft document.
7. PG&E – V. Madani, “PMU Placement Consideration Factors,” June 9, 2010. [http://www.naspi.org/meetings/workgroup/2010\\_june/presentations/session\\_04/madani\\_pacific\\_g\\_as\\_pmu\\_placement\\_20100609.pdf](http://www.naspi.org/meetings/workgroup/2010_june/presentations/session_04/madani_pacific_g_as_pmu_placement_20100609.pdf)
8. M. Patel, et al., “Real-Time Application of Synchrophasors for Improving Reliability,” NERC Report, Oct. 2010. Available online at <http://www.naspi.org/resources/resources.stm>.

- Digital Relays (DR) are asset protection devices that analyze power system voltages and currents to detect faults in an electric power system and use advanced logic to determine whether the relay should trip to protect the line or equipment behind it. Digital relays are capable of taking and recording synchronous phasor measurements.
- Dynamic Performance Recorders
  - Phasor Measurement Units (PMU) measure electrical currents and voltages, and calculate current- and voltage-phasors time-synchronized against the GPS/UTC clock. PMUs and some DSRs are capable of continuous storage.
  - Dynamic Swing Recorders (DSR) capture power system dynamic events such as short-term and long-term low frequency oscillations and abnormal frequency or voltage excursions.

Thousands of PMUs, DFRs, DRs, DSRs, SOERs and DFRs exist in the North American electric grid, many installed in the wake of the 2003 blackout -- and more are being installed with the growth of transmission smart grid investments. The information they provide is used for fast decision making and control (DRs), and off-line analysis (DFRs, DSRs, and SOERs). This paper will focus on the placement of PMUs, but the placement considerations discussed here apply to in most cases to placement of all the above dynamic grid condition monitoring devices.

## 1.2 Phasor data applications

Although PMU data collected at a few points on the grid can reveal conditions across a wide area and inform a variety of grid applications, PMU placement should support the needs and functionalities of the intended applications and system characteristics. As a result, the discussion on PMU siting is organized according to real-time and off-line applications with sub-categories as shown in Table 1. PMU siting recommendations for these applications are provided in Section 2.

**Table 1. Synchrophasor data applications**

<b>Real-Time Applications</b>	<b>Off-Line Applications</b>
Visualization and situational awareness <ul style="list-style-type: none"> <li>● Situational awareness</li> <li>● Generating stations</li> <li>● Flowgates and regional transmission interfaces</li> <li>● Separation islands</li> </ul>	Analysis and Assessment <ul style="list-style-type: none"> <li>● Baselining and correlation analysis</li> <li>● Disturbance analysis</li> <li>● Model validation</li> <li>● Frequency response analysis</li> <li>● Renewable generation</li> </ul>
Monitoring and alarming <ul style="list-style-type: none"> <li>● Phasor-data augmented state estimation</li> <li>● Phasor-data only state estimation</li> <li>● Small-signal stability monitoring</li> <li>● Voltage stability</li> <li>● Thermal monitoring and congestion monitoring</li> </ul>	System planning <ul style="list-style-type: none"> <li>● Load characteristics</li> <li>● Primary frequency response</li> </ul>
Protection and control <ul style="list-style-type: none"> <li>● Out-of-step protection</li> <li>● Small-signal stability protection</li> <li>● Long-term stability control</li> </ul>	

## 2. Siting Recommendations

This section discusses PMU placement for each application areas, with more technical details given in the appendices. It offers analytical considerations behind various siting approaches for selected applications. Those who seek more details on synchrophasor applications should study the RAPIR report [48] and other resources at [www.naspi.org](http://www.naspi.org).

Unless the text specifically indicates otherwise, the discussion below presumes that every installed PMU is networked through a high-speed communications system to deliver its collected data in real-time through one or more phasor data concentrators to applications that will use its time-synchronized data and archive it for later analysis.

### 2.1 Real-Time Applications

#### 2.1.1 Visualization and Situational Awareness

Situational awareness is the challenge of providing relevant operational information for a wide area of the power system, in comprehensive and usable ways, to grid operators and engineers in real-time. Desirable situational awareness functionalities include:

- Visibility into conditions across multiple control areas, if not the entire interconnection
- Situational awareness dashboard
- Real-time compliance monitoring with reliability baselining (angle of separation, voltage and angle profiles, MW, MVAR flows, load-resource imbalance)
- Frequency stability and islanding

Desirable PMU locations for situational awareness include:

- Generation stations with a combined capability above a certain MW level, such as 500 MW and above.
- Key substations in the high-voltage (230 or 345 kV and above) transmission system, including those with HVDC stations, FACTS controller installations, or large tap-changing and phase-shifting transformers
- Major load centers
- Flowgates requiring transmission loading relief in the last three years
- Significant regional and interregional transmission interfaces
- Pre-defined separation islands.

At the end of this paper, we offer recommendations for the synchrophasor project manager who has more potential PMU locations than funds, and must choose where to site a limited number of PMUs to serve numerous phasor data applications.

##### 2.1.1.1 Situational awareness

Each PMU should measure the voltage phasors on the substation bus, and one or more line current phasor measurements. In general, wide-area visibility across the power system improves when more voltage and current phasors are available (i.e., when there is broad coverage by more PMUs across the region). It would be desirable to measure transmission line currents with substation-based PMUs with multiple voltage and current channels. Broad regional coverage using multiple-channel PMUs provides

redundancy and better visibility in the event that individual PMUs or substations go down for maintenance or become disabled.

It is preferable to have a more evenly distributed placement of PMUs across a control region, rather than a high concentration in a small region. More discussion on this PMU coverage will be provided in conjunction with the State Estimation topic in Appendix A.

#### **2.1.1.2 Generation stations**

In a generation station, it is desirable to measure all the line currents (including the step-up transformer) and both the high-side and low-side voltages. In situations where it is not practical to locate a PMU directly at a generator substation, PMUs can be placed on neighboring buses, with current phasor measurements on the connecting lines. Another possibility is to measure a cluster of buses around the generating station and use appropriate weights to interpolate the voltage phasor at the generating substation. An example is the Browns Ferry Bus voltage angle, the reference angle for the Eastern Interconnection, which is computed from the bus voltage phasors in three locations.

Because changes in the status of large generation stations (1,000 MW and above) tend to be significant contingency events, PMUs should be placed at the substations of large nuclear power plants, coal plants and single or aggregated wind generators.

#### **2.1.1.3 Flow gates and regional transmission interfaces**

The PMU locations for load centers, flowgates, and transmission interfaces may overlap. Load centers are likely to be at the receiving end of flowgates and transmission interfaces. Monitoring of these functionalities may require multiple PMUs. For example, a load center may have several in-feeds, and the number of PMUs needed will be based on the dependency of the flows coming from the those in-feeds. Similarly, for flowgates and transmission interfaces, it may be necessary to measure both the sending and receiving ends. PMUs at intermediate points of a power transfer path would also be desirable if there are significant inflow and/or leakage. Furthermore, multiple PMUs are needed if the flows on parallel paths have a high degree of independency.

#### **2.1.1.4 Separation islands**

PMU locations for system separation should reflect the control area's system separation planning. These separation points are often at interregional and transmission interfaces. It may be appropriate to install PMUs on both sides of a separation interface, to monitor the viability of an island, restoration progress within an island, and readiness for resynchronization.

### **2.1.2 Monitoring and Alarming**

Wide-area and local monitoring and alarming are specific situational awareness applications. Desirable monitoring and alarming functionalities include:

- Real-time performance monitoring and trending
- Anomaly characterization and alarming (real-time alarming on hard limits and “out of normal” conditions, suggesting preventive actions)
- State estimation
- Small-signal stability monitoring (inter-area modes 0.1-1 Hz)
- Small-signal stability monitoring (local inter-plant modes 1-3 Hz)

- Voltage stability monitoring and assessment
- Thermal monitoring (overload).

Real-time performance monitoring and anomaly characterization require the processing of historical PMU data to establish a range of normal operating conditions. PMU location requirements are similar to those for situational awareness.

### **2.1.2.1 Phasor-data augmented state estimation**

PMU data can be used to augment a conventional ICCP-data based state estimator, forming a phasor-assisted state estimator. PMU data, with known voltage and current angles, may improve the fidelity of a conventional state estimator [1-3]. Furthermore, if PMUs are placed at the critical buses<sup>2</sup>, the convergence of SE solutions will improve [4]. Additional discussion is provided in Appendix A.

### **2.1.2.2 Phasor-only state estimator**

It is also possible to construct a phasor state estimator (PSE) using only synchrophasor data. In the initial stages of PMU installation in a power control region, it is realistic to implement such an estimator only for the high-voltage transmission system. To implement PSE for  $N$  interconnected buses, a minimum of  $N/2$  line current phasor measurements with corresponding bus voltage phasor measurements is needed. Furthermore, if angle-bias correction capability is required, then a minimum of  $(N-1)$  line current phasor measurements is needed. If the robustness of the PSE with respect to the loss of any one PMU or transmission line (i.e., zero line current), additional PMUs will be needed. More details on PSE are provided in Appendix A.

### **2.1.2.3 Small-signal stability monitoring**

Power system swing mode stability monitoring can be achieved using a variety of signals, including bus voltage magnitudes, bus frequencies, line current magnitudes, and line active power flows. Bus voltage angles can be used if they can be compared to a reference angle. Local and intra-area modes can be monitored using transmission line PMU data within a control area. Inter-area mode can be detected by tracing lower frequency modes through several other control regions. Techniques such as Prony analysis, eigensystem realization algorithm (ERA), and matrix pencil methods [5-7], can be used to extract the frequencies and damping of the interarea modes and their mode shapes from system transient response following a large disturbance.

Some HVDC systems and FACTS controllers are equipped with inter-area mode damping controllers, and thus a PMU measuring the modulating signals (such as a current or power injection) can be useful in validating the performance of the damping controller. For PMUs located at load centers and generating stations, the line current magnitude or bus frequency would in general have better oscillatory contents than the voltage magnitude because of load sensitivities to voltage variations and control actions from generator voltage regulators. Ambient or “blue sky” PMU data can also be used to analyze the oscillatory frequencies in a power system using the Welsh or Yule-Walker techniques [8-10]. A more detailed discussion is provided in Appendix B.

### **2.1.2.4 Voltage Stability**

---

<sup>2</sup> If the ICCP measurements on a critical bus are not available, the state estimator will become unobservable.

Several control areas have implemented voltage stability monitoring using conventional state estimators. Synchrophasor data can add to this capability on a real-time basis or by improving the quality and convergence of the state estimator solution. If PMUs can be placed at load centers and on transmission interfaces, a voltage stability analysis can be performed using various methods [11-15]. Because of the high-sampling rate, PMU data can be used to trace a larger segment of the voltage stability curve, more commonly known as the *PV*-curve. Furthermore, the tap positions of large transformers can be monitored for voltage stability. More discussion on voltage stability monitoring can be found in Appendix C.

#### **2.1.2.5 Thermal monitoring and congestion management**

For thermal monitoring and congestion management of overhead transmission lines, it is necessary to have PMUs measuring voltage and current phasors on the line. It would also be useful to monitor the local ambient temperature and wind speed, unless real-time meteorological information can be obtained separately. Effective use of phasor data for dynamic line loading and congestion management on key transmission lines or assets requires one or more PMUs directly monitoring the targeted assets.

### **2.1.3 Advanced Applications – Protection and Control**

PMUs with high sampling rates matched by a low-latency communications network, can provide synchrophasor data for protection and control actions. Desirable protection and control functionalities include:

- Short-term stability control (e.g., transient stability)
- Small-signal stability protection and control
- Long-term stability control (e.g., wide area frequency and voltage stability)

At the risk of stating the obvious, a PMU intended to provide protection and control should be located close to the asset it is intended to protect or control, and be partnered with one or more reference PMUs and PMUs at any devices that interact with the target asset for control purposes [23].

#### **2.1.3.1 Out-of-step protection for short-term stability control**

Short-term stability control seeks to retain the stability of a power system under transient conditions. Out-of-step relays are generally used to detect increased separation between two target buses or areas. Distance relays and timers are employed to track the apparent impedance with respect to some preset regions. [20,21] With synchrophasors, the separation angle between two locations in a power system can be directly measured to operate the required relaying to maintain system stability. An added benefit of such a relay scheme is that the settings can be made adaptive to reflect system loading conditions. To apply such synchrophasor out-of-step relaying scheme, a thorough contingency analysis needs to be performed to determine the required set of PMUs and their locations [22]. A DOE funded project on advanced wide-area warning system with adaptive protection is described in [23].

#### **2.1.3.2 Small-signal stability protection and control**

The goal of small-signal stability protection is to use phasor data to identify instances when small-signal oscillations are occurring, determine whether those oscillations are damped quickly or swinging out of control, and initiate actions to stabilize frequency. A PMU-based wide-area monitoring and situational awareness system can identify small-signal oscillatory events and determine whether an oscillatory problem is under way. Long records of synchrophasor data on both events and blue sky conditions are needed for baselining and event analysis, to determine how to effectively implement stability protection and control measures.

Power swing damping enhancement typically is achieved using local signals as inputs to the controllers. For example, the modern dual-input power system stabilizer uses the machine speed and the electrical power output as the input signals. However, local signals may not have sufficient inter-area mode content to be used as an effective signal for interarea mode damping. To create a more inter-area-rich signal, synthesized remote angle signals have been proposed in [24] for an inter-area mode damping control design of a thyristor-controlled series compensator (TCSC). By placing PMUs at centers of remote groups of machines, the aggregated angles can be directly measured without the need of estimating some equivalent impedances. On the other hand, such control implementation requires the compensation of latency in phasor data computation and computer communication networks [25,26].

### **2.1.3.3 Long-term stability control**

Synchrophasor data can be useful for system frequency and voltage control. The Tokyo Electric Power Company (TEPCO) has implemented an islanding scheme to balance the active and reactive power in the islanded system by load shedding [27]. Entergy has used phasor data for system restoration after a hurricane disrupted system operation [28].

Synchrophasor data can also be used to detect voltage instability and to dispatch of reactive power for system reliability. By improving the reliability of the state estimator, contingency analysis can be enhanced, which is beneficial for voltage control.

## **2.2 Off-Line Applications**

### **2.2.1 Analysis and Assessment**

System analysis and assessment of system reliability based on PMU data are off-line functions intended to establish secure operating ranges for real time applications and validation of simulation models and parameters for operations and planning studies. Desirable analysis and assessment functionalities include:

- Baseline normal phase angle trends
- Pattern recognition and correlation analysis
- Disturbance analysis compliance
- Model validation
- Frequency response analysis
- New applications evaluation and limits definition
- Phasor network performance monitoring and data quality

Desirable PMU locations include those for situational awareness and monitoring and alarming as well as:

- Generating stations equipped with power system stabilizers
- Controllable devices such as HVDC systems and FACTS controllers
- Variable generation such as areas with significant concentrations of intermittent resources (e.g., wind farms)

#### **2.2.1.1 Baseline and correlation analysis**

Establishing a baseline for normal phase angle trends requires, among others, archiving the relative bus voltage angle difference between certain locations within a control region and across control regions.

These locations are most likely to be either dominant generation sites or load centers -- in other words, PMU locations that parallel high-priority sites for situational awareness. The angular difference would also be a strong function of the system loading. Such analysis can be aided by off-line studies and sensitivity analysis. The results of trending analysis can be useful in recognizing unusual operating patterns in a power system. In particular, the outage of a transmission line with high loading will disrupt the normal power flow pattern and increase the loading on parallel lines and hence the angular separation between generation and load.

#### **2.2.1.2 Disturbance analysis**

Analysis of large disturbances has been performed quite extensively in the U.S. Western power system using PMU data. High-speed PMU data are ideal for capturing the initial power transients at the onset of a disturbance and studying the subsequent inter-area power swings. The analysis can expose vulnerabilities of a power system not captured in simulation programs, whose models and parameters may not reflect exactly the real system. The post-mortem analysis of Western system breakup in the 1990's led to the enforcement and monitoring of PSS operation to reduce inter-area oscillations. Having PMUs on generating stations can verify the effectiveness of their PSS tuning and control action.

#### **2.2.1.3 Model validation**

Generators and other control equipment such as FACTS controllers have significant impact on system stability. Thus it is important that they be represented properly in simulation programs. By installing PMUs on the bus bars at the substation and measuring the voltage phasor and the output current phasor, the machine response due to a disturbance can be simulated. If it is not possible to put the PMU right at the generating station, then PMUs located at neighboring buses may be used to calculate pseudo-PMU measurements for the generation bus. Such parameter testing is not as comprehensive as staged generator tests [16] which require taking the generator off-line. Disturbance-based parameter validation can be performed without taking the generator off-line, and its performance can be improved if some internal variables of the equipment (such as the generator speed and field current) can also be recorded. A more detailed discussion is given in Appendix D.

#### **2.2.1.4 Frequency response analysis**

Frequency response analysis of power systems involves injecting a probing signal in one location and measuring the response at other locations [17-19]. In the U.S. Western system, the probing signal can be either the pulse application of the Chief Joseph braking resistor and set-point pulsing of the Celilo HVDC current injection. Thus PMUs must be located at the signal injection point, as well as at all the monitoring locations. Signal processing techniques can be used to derive the frequency response from the injection point to the monitored locations. A particular benefit of such frequency response analysis is to detect undesirable changes in interarea mode damping.

Practically speaking, as long as there is a PMU located at the point where the probing signal initiates, frequency response to the probing signals can be monitored for later analysis using any or all of the PMUs located across the interconnected system.

#### **2.2.1.5 Renewable generation**

Penetration of renewable generation from wind, solar, and other intermittent sources, is growing within U.S. and other power systems. The intermittency of such renewables increases the generation and load balancing challenge in power system dispatch and operation. Placing a PMU at the collection point of a cluster of variable generators can give operators early tracking of the output of these intermittent

resources, while planners can use collected historical data to better understand the generators' production patterns and response characteristics. Currently the energy source condition (wind speed, insolation level, etc.) are not collected by PMUs. However, if such information is collected by spare channels in a PMU or via other means, the data set as a whole would improve off-line and control room renewable generation analysis and forecasting.

## 2.2.2 System Planning

Many issues relevant to system planning have already been discussed above, such as baselining, model validation, stability, and control. In this section, we will discuss some additional topics, including event analysis, load characterization, islanding, and primary frequency response.

### 2.2.2.1 Load Characterization

Realistic load models are important for off-line stability studies, but are difficult to obtain. As a result, conventional stability simulation studies mostly use constant current to model active power loads and constant impedance for reactive power loads. Commercial simulation programs allow a user to select a proportion of load to be of induction motor type. In addition, electronic load is growing whereas incandescent lighting (resistive load) is decreasing. However, load simulation programs do not often reflect the changing nature of power loads with respect to changes in the electrical behavior and penetration of modern end use devices. While PMUs placed at load centers will not reveal changes in the makeup of loads, they can reveal changes in the electrical characteristics and behavior of aggregated loads.

Recently there are significant interests in studying the phenomenon of fault-induced delayed voltage recovery (FIDVR). For example, Southern California Edison has successfully used phasor data to calibrate load models. Going forward, for power companies concerned about their load models, PMUs should be installed at appropriate sub-transmission level substations so that the load sensitivities to system frequency and voltage changes can be monitored, not only during large disturbances, but also smaller events like nearby capacitor bank switching.

### 2.2.2.2 Primary Frequency Response

Synchrophasor data, with its high sampling rate, provide a more tracing of system frequency after major generation and/or load trips. In fact, one of the early uses of phasor is its use in computing the primary frequency response  $\beta$  of the generators using the expression

$$\beta = \frac{\text{amount of generation lost}}{\text{frequency deviation}} \text{ MW}/0.1 \text{ Hz}$$

The variation of  $\beta$  in the eastern US power system has been tracked for many years. The functionality should continue to ensure that there is enough primary frequency response to deal with loss of multiple generators. Any PMU placed on the high voltage transmission levels can be used for tracking frequency response. However, those PMUs close to major power plants, HVDC systems, and loads, may be able to provide more effective monitoring of the frequency response capability of the installed equipment.

### 3. PMU Location Selection

The siting guidance in this paper can help to determine the number and location of PMUs required for implementing desired applications and functionalities. In case of staged deployment, in which a fixed number of PMUs will be installed at various stages, a ranking of the substations to satisfy the desired functionalities should be performed. Weights can be applied to each of the relevant criteria to provide some further separations in scores [47]. Substations on the Critical Cyber Asset (CCA) list and at higher voltage levels suitable for multiple criteria should be preferred over other locations. In some cases, the cost of installing PMU at substations may be factored into considerations, particularly for those substations without adequate communication infrastructures.

The synchrophasor project manager should start a PMU placement exercise with the intention that every PMU installed should serve multiple applications – thus, the highest value PMU locations are likely those at the asset owner’s most critical substations and flowgates, followed by its largest power plants and load centers. If the project manager has numerous PMUs deployed already, then the next PMU could be placed to serve a specific and limited purpose – for instance, at the point of interconnection for a large new wind development area, to facilitate its operational integration. On the other hand, if the project manager has only a few PMUs installed, then the next PMU should be placed in a location that fills in a gap across the region’s electrical network topology, to improve visibility across the entire control area or region (which may be an important substation or interface).

Some additional considerations that may affect PMU placement decisions (or the timing of when a PMU is placed at a particular location) include:

- Does the application that the PMU serves require real-time phasor data delivery? If so, then the cost of PMU installation will require access to a high-speed communications network. This tends to focus early PMU placements at substations that already have such network connections in place.
- Will the PMU’s data feed a mission-critical application (such as real-time automated voltage management) rather than an informative but redundant application (such as wide-area situational awareness)? If the application is mission-critical, then the asset owner will probably view the PMU as a critical asset and a critical cyber asset, and should place the PMU inside a substation or another site that already has an established physical security and cyber-security perimeter.
- If the project manager needs to get PMUs installed quickly and inexpensively, he or she should look for substations that are coming up for scheduled maintenance or upgrade; it is often easier and less costly to add PMU installation into an upcoming substation redesign and construction process than to handle the PMU and communications network installation as a stand-alone project.
- For PMUs that serve multiple applications (such as wide-area situational awareness, generator monitoring and voltage management), it’s advantageous to site these PMUs at substations that receive priority attention for equipment and communications network maintenance, to assure that these PMUs sustain high availability and performance. This is a particularly important consideration for a “reference PMU” that is used as the point against which phase angles are calculated from other locations.
- A project manager for one utility may want to coordinate PMU placement with his or her neighbors. By working together to coordinate PMU siting across the region, they may be able to improve coverage and observability across the entire region – and better insight into their own control areas – using fewer PMUs.

### 3.1 A Sample Selection Matrix

An approach that could be useful in practice is to select the real-time and off-line applications discussed above and assign weights to the substations in the system according to their relevance to provide PMU data for those applications. The substations with the highest weighted sum would be most suitable for installation of new PMUs.

Table 2 is an illustration of this approach, in which three application categories are selected as important for a particular control region. Each of the categories carries a maximum of 10 points. In this example, the Cardinal substation comes out on top because it is important in all three categories.

Table 2. Using weights to prioritize the relevance of PMUs at substations

PMUs at Substations	Situational awareness	Voltage stability	Load monitoring	Total
Cardinal	10	5	5	20
Orioles	5	5		10
Blue Jay	3	2	4	9

## 4. Conclusions

This white paper offers some guidelines for designers and users of the synchrophasor systems to decide where to place new PMUs. The paper addresses synchrophasor data applications and their PMU data requirements to be considered as part of the PMU placement process. Both real-time and planning related applications are addressed. In the appendices, PMU placement rationales and processes for some of the applications are explained in more details. As background materials, this paper has provided a list of documents developed by various control areas on their PMU siting and application criteria.

## Appendix A: State Estimation (SE)

### A.1 Phasor-Assisted SE

Synchronized phasor data of bus voltage and line current measurements can be added to conventional state estimation [1] to improve the performance of the state estimation. Results on phasor-assisted SE have been reported by Sevillana de Electricidad (CSE) [29] and San Diego Gas & Electric [30]. The CSE SE application involves a total of 309 SCADA measurements with 23 phasor angle measurements. The San Diego Gas & Electric [30] approach includes the measured angles in the form of angle differences between phasor measurement units to avoid setting a PMU bus as reference. Several other modifications to this approach have been pursued, and are reported in [31-33]. In these implementations both voltage and current phasors (magnitude and angle measurements) are incorporated into the measurement vector.

A “Power Conversion” approach has been used by the British Columbia Transmission Corporation (BCTC) [34]. In this approach the voltage and current phasors are used to compute complex power from the phasor measurements at each branch measured by the PMU. The complex power is decomposed into active power and reactive power, which are used as pseudo-measurements in the SE.

For companies with limited PMU deployment or in a staged deployment process, the observations in [4] are helpful:

- a. PMU measurements on critical buses are most useful. If the ICCP data on critical buses are not available, but the PMU data are, then the observability of the state estimator is preserved.
- b. If station-based PMUs<sup>3</sup> are used, then preference should be given to buses with connections to many other buses.
- c. Bad data detection for phasor-assisted SE is improved by placing PMUs in all buses with critical measurements, allowing bad data in the system to be detectable.

Note that there is no requirement for the minimum number of PMUs to be installed for phasor-assisted state estimation.

### A.2 PMU-Only SE

With bus voltage and line current phasor measurements, voltages at neighboring buses connecting to PMU buses can be calculated directly without iterations [1]. As an illustration, consider a 4-bus radial network with 2 PMUs in Figure 1, with voltage phasor measurements on Buses 1 and 2, and the line current phasor measurements on Lines 1-3 and 2-4. Knowing the line parameters, the voltages on Buses 3 and 4, and the line current on Line 3-4 can be calculated without using any iteration. In this case, the 4-bus system is completely observable given the 2 PMUs.

Using graph theory, it can be shown that for an interconnected power network with  $N$  buses, a minimum of  $N/2$  voltage and line current phasor measurements for  $N$  even and  $(N+1)/2$  for  $N$  odd, regardless of the number of lines in the network, in order to achieve observability. The 4-bus system is an illustration of this idea. If relay-type PMUs measuring only 1 voltage phasor and 1 current phasor are used, then  $N/2$  ( $(N+1)/2$  if  $N$  is odd) PMUs are needed. If PMUs capable of measuring all the line and transformer currents at a substation are used, only 1/4 to 1/3 of the buses need to be equipped with PMUs [4].

---

<sup>3</sup> A station-based PMU measures the currents on all the line connected to the substation.

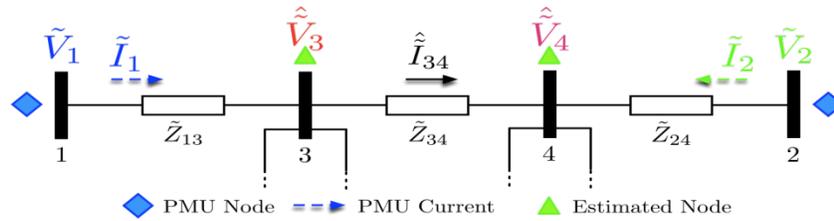


Figure 1: Four-bus System

More recently there is a need to ensure high quality phasor data used for phasor-data based applications. An approach to ensure phasor data quality is to install additional PMUs so that their measured data can be cross-checked. Figure 2 illustrates that that voltage phasor on Bus 3 can be calculated from either the PMU on Bus 1 or the PMU on Bus 2. This provides a mechanism to validate the measured phasor data. An algorithm has been developed in [35,36] in which phase errors in synchrophasor measurement data can be corrected.

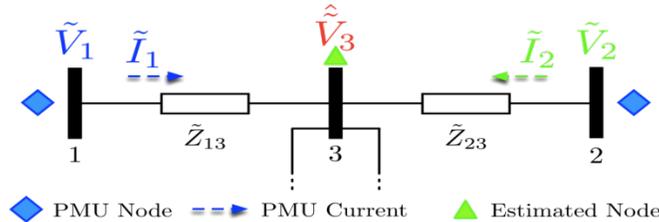


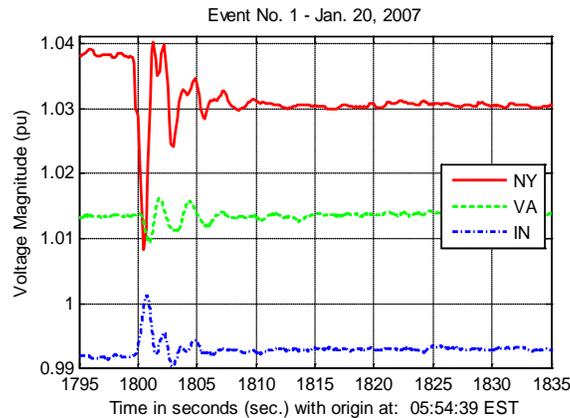
Figure 2: Observable Island with Redundancy for Phase-Bias Correction

It can be shown that in order to apply the phase error correction method in [36], for an interconnected power network with  $N$  buses, a minimum of  $N-1$  line current phasor measurements is needed, regardless of the number of lines in the network. The 3-bus system in Figure 2 is an illustration of this idea. If relay-type PMUs measuring only 1 voltage phasor and 1 current phasor are used, then  $N-1$  PMUs are needed. If PMUs capable of measuring all the line and transformer currents at a substation are used, then about 1/2 of the buses need to be equipped with PMUs [36].

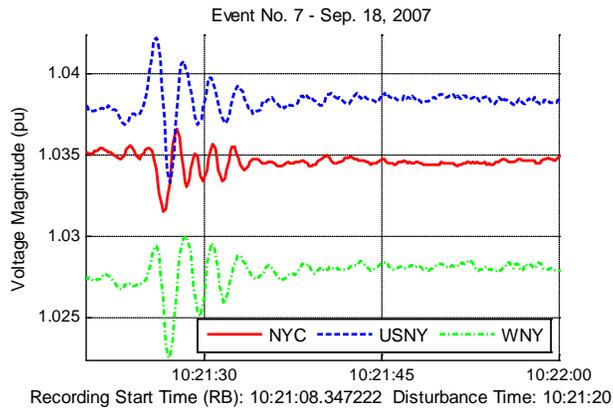
## Appendix B: Modal Oscillations

### B.1 Modal Oscillation Identification

Power transfer interfaces and transmission buses connecting neighboring power controls areas are typically strategic locations for PMU placement for observing and identifying oscillations. After the initial onset, the subsequent propagation of a disturbance can be seen in the resulting oscillations of voltage and current measurements at strategic locations [37]. In certain locations, such as generator control buses and load centers, the voltage and current contents can be distorted by local control actions and load responses. The locations to best see system oscillations are transmission buses with no nearby generators and loads.



**Figure 3: Voltage responses in three regions**



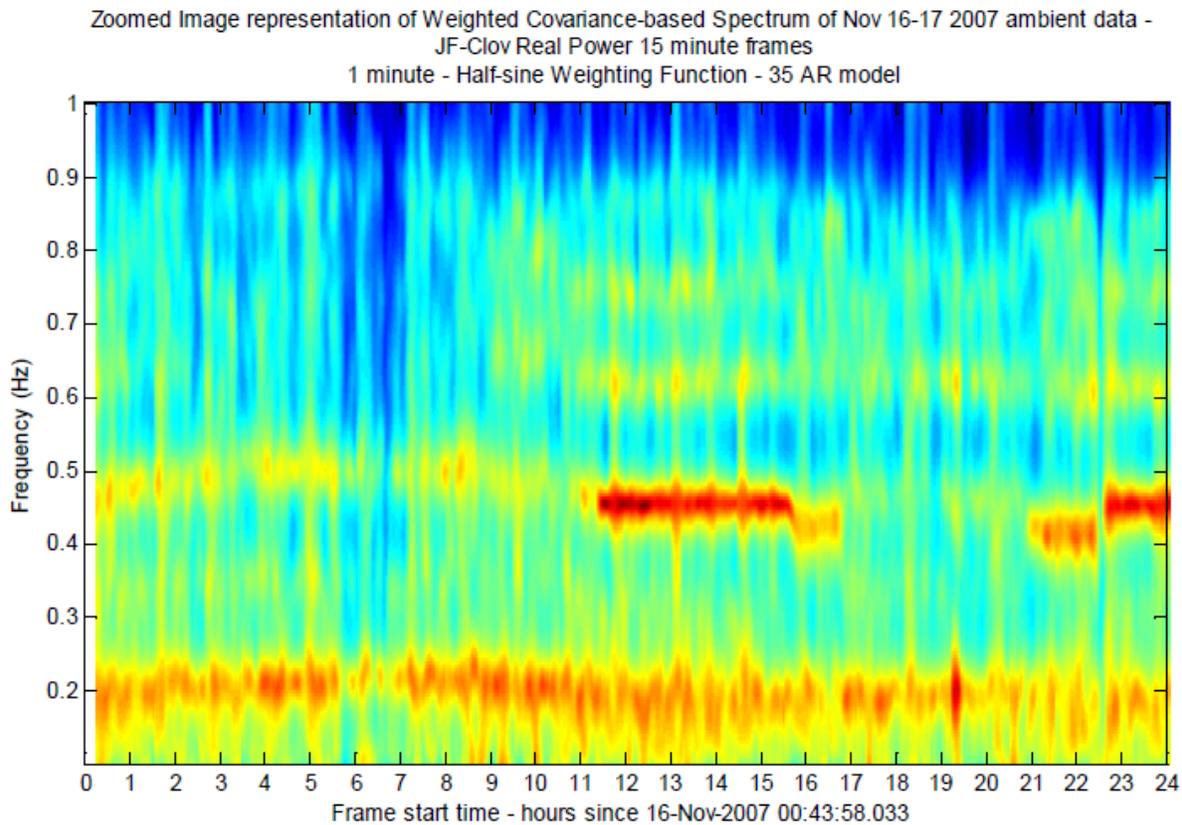
**Figure 4: Voltages in NY City, Upstate NY, and Western NY**

To illustrate consider the measurements in Figure 3. The plot shows the voltages on a New York transmission bus close to the disturbance, a transmission bus in Virginia far from the disturbance, and a generator bus in Indiana. The NY bus voltage shows large oscillations, with the local voltage regulation by a STATCOM visible (the dip when the voltage goes high). The VA bus voltage shows clear oscillations, but with smaller amplitude. In addition, the voltage oscillations at the NY bus and the VA bus are in phase, although these two buses are several hundred miles apart. The IN bus voltage behaved quite differently, as the excitation system of the generator at the bus reacted to boost the bus voltage, in order to provide more reactive power to the neighboring buses.

Figure 4 shows the voltage responses for several buses in New York in a Northwest region islanding event. The upstate NY (USNY) bus and the western NY (WNY) bus, both transmission buses, have similar voltage responses, exhibiting single modal responses. The NY City (NYC) bus, however, has a very different voltage response, with multiple modal responses. This is not unexpected, given the large amount of load in the metropolitan area, which can have very nonlinear dynamic behaviors.

As noted in Section 2.2.3, methods for obtaining modal frequencies and damping from disturbance response include Prony, ERA, and matrix pencils.

For ambient data, the Welch periodogram and the Yule-Walker method can be applied. The “mode meter” algorithm developed by John Pierre and Dan Trudnowski for ambient data analysis has been used in BPA and SCE. A 24-hr PMU data time-frequency analysis plot is shown in Figure 5.



**Figure 5: Ambient analysis of a line current magnitude in the AEP system**

## Appendix C: Voltage Stability

Voltage instability is characterized by inadequate reactive power supply at certain locations in a power network after disturbances, and/or as a result of gradual increment of the load. The dynamics of voltage instability are slow as compared to small signal or transient stability, but some phenomena lasting seconds are also of interest; hence, high-sampling rate phasor measurements can provide additional information for voltage stability monitoring.

Real-time PMU data can be used to supplement voltage stability monitoring by generating the so-called Power-Voltage (*PV*) curve characteristics. Consider the system in Figure 6, in which a load cluster (receiving end) is connected to a generation cluster (sending end). In such cases, the classical Thévenin equivalent approach can be used to perform a simplified voltage stability analysis [38], [39], [12]. The Thévenin equivalent of the system in Figure 6 is shown in Figure 7. With this approach, the sending end is assumed to be a stiff network and can be represented by a constant voltage source behind an equivalent impedance. The measurements are usually placed at the load bus.

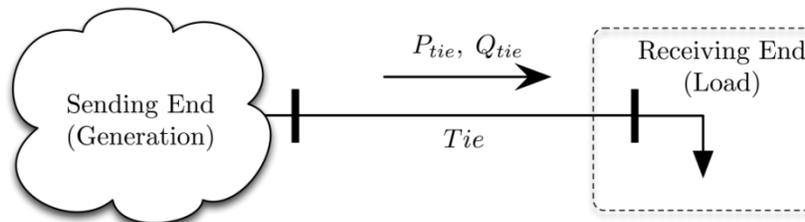


Figure 6: Sending and receiving areas of a power system interconnection

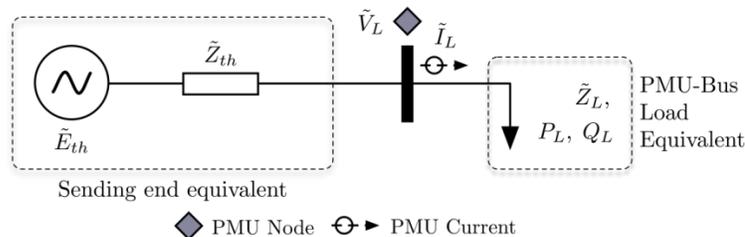
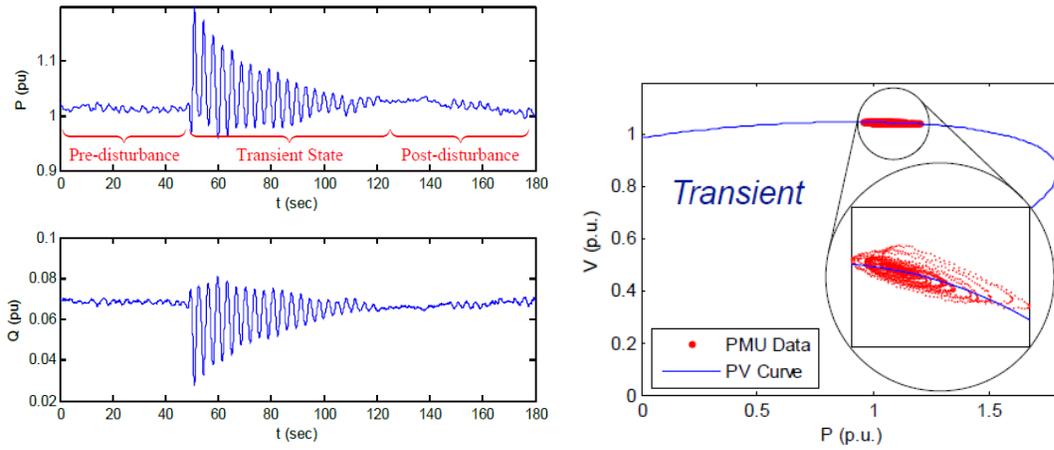


Figure 7: Thévenin Equivalent for Voltage Stability Analysis

Using the PMU voltage and current phasors, it is possible to estimate the parameters of the equivalent circuit in Figure 7 [40], [41], [13], [14]. Once the parameters have been identified, voltage stability margin can be assessed using, for example, the *PV* curve of the network or calculating a voltage stability index. Note that with only one PMU, time synchronization is not an issue. There are several methodologies to determine the equivalent network parameters, and most of these methodologies involve using a window of data in their estimation.

If high-sampling rate phasor data are used, then line active power flow and voltage magnitude excursions during a disturbance may generate a substantial segment of the *PV* curve. In [13], a window of phasor data from an oscillatory event was used to construct the Thévenin equivalent, which is then used to

generate the PV curve. Figure 8 shows the measured PMU data superimposed on the calculated PV curve. The PMU data was sampled at 30 points per second, and a 3-minute window was used.



**Figure 8: Phasor Measurements at a bus of the receiving end of the Network and the estimated PV curve for the transient portion of the data [13]**

## Appendix D: Model Identification and Verification

Identification of power system model parameters is an important practice that helps keeping planning models up-to-date [42]. This is of interest because dynamic models are used in the planning stage to obtain realistic scenarios for system expansion. The models may also be used for on-line applications as discussed in Section 2. Here we discuss some of the methodologies used for identifying power system device models and some of the requirements regarding placement and additional measurements needed by the techniques.

### D.1 Calculating Pseudo PMU Measurements at Generator Bus

Although it is desirable to site a PMU at generating stations with more than 500 MW of installed capacity, the substation may not be owned by the TO, and thus another way needs to be found. Consider the connections in Figure 9, in which a PMU cannot be placed at the nuclear generating substation. If a PMU is placed at Bus 1 and a second PMU is placed at Bus 2, then the generator terminal bus voltage phasor and the terminal current phasor can be computed, which are referred to as pseudo PMU measurements.

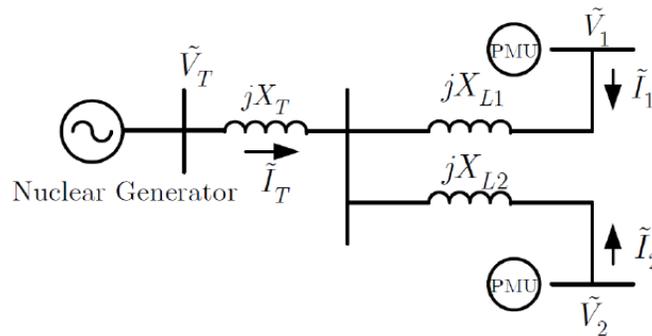


Figure 9: Calculating pseudo PMU measurements at a nuclear generator bus

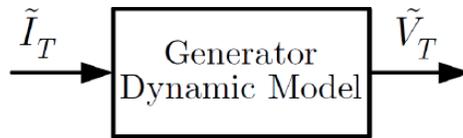
This situation in Figure 9 is also applicable to wind farms tapped into high-voltage transmission lines.

### D.2 Identifying Dynamic Model Parameters

A common procedure in generator parameter identification is to perform a variety of tests with the generator off-line, typically during the annual maintenance period. An example of such tests is discussed in [43] for a Hydro Unit in the TVA system. The paper provides a diagram of the set-up of a PMU to monitor all desired measurements in the standard IEEE testing guidelines [44,45]. Measurements of all stator voltages and currents were made using the PMU. In addition, the field voltage, the field current, and the generator rotor speed are also measured. The parameter identification was performed using the trajectory sensitivity method<sup>4</sup>, which is applicable to both linear and nonlinear models. In [46], the trajectory sensitivity method is applied for the identification of nonlinear excitation systems models. Use of these techniques requires the data to have sufficiently rich information, i.e., the measurements must show the effects of the parameters to be identified. For example, to identify excitation system voltage limits, the impact of the limit must be visible in the measured variables.

<sup>4</sup> Other methods for model parameter identification include Kalman filtering.

For generator parameter identification, a PMU can be used to measure the terminal voltage and current phasors either on the low-side or high-side of the step-up transformer during a disturbance. If possible, it is also desirable to measure the generator field variables and speed (typically from the front standard speed sensor on the turbine side). For a disturbance close to the monitored generator, the phasor data can be used to identify those generator model parameters that participate in the disturbance. This idea is shown in Fig. 10. If the generator model and its parameters are correct, using the measured current phasor as input to a generator model will result in the generator output terminal voltage to be identical to the measured value. If this is not so, trajectory sensitivities can be used to tune the relevant parameters.



**Figure 10: Input-output generator dynamic model block diagram**

Note that generator parameters that can be identified must be consistent with the sampling rate of the synchrophasor data rate. For example, to identify subtransient model parameters, the sampling rate needs to be 100 – 200 Hz, higher than the 30 Hz sampling rate currently proposed.

It is also of interest to monitor the steady-state and dynamic model characteristics of renewable resource models such as wind turbines, solar cells, flywheels, and SMES.

**Acronyms**

PMU	Phasor Measurement Unit
SE	State Estimation
CSE	Compañía Sevillana de Electricidad
TVA	Tennessee Valley Authority
NYISO	New York Independent System Operator
SOE	Sequence of Events
RTU	Remote Terminal Unit
DCS	Digital Control System
DR	Digital Relays
DMU	Dynamic Measurement and recording Units

**Glossary**

PMU Bus	A bus with PMU measurements of voltage and all available currents incident to the bus
Non-PMU Bus	A bus without PMU measurements
Line-based PMU	A PMU measuring one bus voltage phasor, and only one current phasor

## References

- [1] J.S.Thorp, A.G. Phadke, and K.J. Karimi, “Real Time Voltage-Phasor Measurement For Static State Estimation,” *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-104, no.11, pp. 3098-3106, Nov. 1985
- [2] B. Fardanesh, “Use of Phasor Measurement in a Commercial (or Industrial) State Estimator,” EPRI, Palo Alto, California, Final Report 1011002, July 2004.
- [3] Eastern Interconnection Phasor Project (EIPP), “Metrics for Determining the Impact of Phasor Measurements on Power System State Estimation,” prepared by KEMA, March 2006.
- [4] J. Chen and A. Abur, “Placement of PMUs to enable bad data detection in state estimation,” *IEEE Transactions on Power Systems*, vol. 21, no. 4, pp. 1608–1615, Nov. 2006.
- [5] J.F. Hauer, C.J. Demeure, L.L. Scharf, “Initial results in Prony analysis of power system response signals,” *IEEE Transactions on Power Systems*, vol.5, no.1, pp.80-89, Feb 1990
- [6] J.J. Sanchez-Gasca and J. H. Chow, “Performance comparison of three identification methods for the analysis of electromechanical oscillations,” *IEEE Transactions on Power Systems*, vol. 14, no. 3, pp. 995-1002, Aug 1999.
- [7] M.L. Crow and A. Singh, “The matrix pencil for power system modal extraction,” *IEEE Transactions on Power Systems*, vol. 20, no. 1, pp. 501-502, Feb. 2005.
- [8] J.W. Pierre, D.J. Trudnowski, and M.K. Donnelly, “Initial results in electromechanical mode identification from ambient data,” *IEEE Transactions on Power Systems*, vol. 12, no. 3, pp. 1245-1251, Aug 1997.
- [9] D.J. Trudnowski, J. W. Pierre, N. Zhou, J. F. Hauer, and M. Parashar, “Performance of Three Mode-Meter Block-Processing Algorithms for Automated Dynamic Stability Assessment,” *IEEE Transactions on Power Systems*, vol. 23, no. 2, pp. 680-690, May 2008.
- [10] D.J. Vowles, M.J. Gibbard, D.K. Geddey, and D. Bones, “Benchmark Testing Methodology for Continuous Modal-estimation Algorithms,” *IEEE PES General Meeting*, pp.1-8, pp. 26-30, July 2009.
- [11] M.M. Begovic, and A.G. Phadke, “Voltage stability assessment through measurement of a reduced state vector,” *IEEE Transactions on Power Systems*, vol. 5, no. 1, pp. 198-203, Feb 1990.
- [12] L. Warland and A. T. Holen, “Estimation of distance to voltage collapse: Testing and algorithm based on local measurements,” in *Proceedings of the 14<sup>th</sup> Power System Computation Conference (PSCC)*, Sevilla, Spain, Jun.2002.
- [13] M. Parniani, J.H. Chow, L. Vanfretti, B. Bhargava, and A. Salazar, “Voltage Stability Analysis of a Multiple-Infeed Load Center Using Phasor Measurement Data,” *IEEE PES Power Systems Conference and Exposition 2006*, pp. 1299-1305, Oct. 29, Nov. 2006.
- [14] S. Corsi and G. N. Taranto, “A real-time voltage instability identification algorithm based on local phasor measurements,” *IEEE Transactions on Power Systems*, vol. 23, no. 3, pp. 1271-1279, Aug. 2008.
- [15] M. Glavic and T. Van Cutsem, “Wide-area detection of voltage instability from synchronized phasor measurements. Part I: Principle,” *IEEE Transactions Power Systems*, vol. 24, no. 3, Aug. 2009.
- [16] P.L. Dandeno, and A.T. Poray, “Development of Detailed Turbogenerator Equivalent Circuits from Standstill Frequency Response Measurements,” *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-100, no. 4, pp. 1646-1655, April 1981.
- [17] J.F. Hauer, W.A. Mittelstadt, K.E. Martin, J.W. Burns, H. Lee, J.W. Pierre, and D.J. Trudnowski, “Use of the WECC WAMS in Wide-Area Probing Tests for Validation of System Performance and Modeling,” *IEEE Transactions on Power Systems*, vol. 24, no. 1, pp. 250-257, Feb. 2009.
- [18] C.E. Grund, J.F. Hauer, L.P. Crane, D.L. Carlson, and S.E. Wright, “Square Butte HVDC modulation system field tests,” *IEEE Transactions on Power Delivery*, vol. 5, no. 1, pp. 351-357, Jan 1990.

- [19] Ning Zhou, J.W. Pierre, and J.F. Hauer, "Initial results in power system identification from injected probing signals using a subspace method," *IEEE Transactions on Power Systems*, vol. 21, no. 3, pp. 1296-1302, Aug. 2006.
- [20] V. Centano, A. G. Phadke, A. Edris, J. Benton, M. Gaudi, and G. Michel, "An Adaptive Out-of-Step Relay," *IEEE Transactions on Power Delivery*, vol. 12, no. 1, pp. 61-71, Jan. 1997.
- [21] J. De La Ree, V. Centano, J. S. Thorp, and A. G. Phadke, "Synchronized Phasor Measurement Applications in Power Systems," *IEEE Transactions on Smart Grid*, vol. 1, no. 1, pp. 20-27, June 2010.
- [22] M. G. Adamiak, A. P. Apostolov, M. M. Bergovic, C. F. Henville, K. E. Martin, G. L. Michel, A. G. Phadke, and J. S. Thorp, V. A. Centano, A. G. Phadke, et al., "Wide-Area Protection – Technology and Infrastructures," *IEEE Transactions on Power Delivery*, vol. 21, no. 2, pp. 601-609, April 2006.
- [23] L. Cibulka, J. Thorp, "Application of Advanced Wide-Area Early Warning Systems with Adaptive Protection," DOE External Peer Review, Oct. 10, 2010
- [24] E. V. Larsen, J. J. Sanchez-Gasca, and Joe H. Chow, "Concepts for Design of FACTS Controllers to Damp Power Swings," *IEEE Transactions on Power Systems*, Vol. 10, pp. 948-956, 1995.
- [25] I. Kamwa, R. Grondin, and Y. Hebert, "Wide-area measurement based stabilizing control of large power systems—A decentralized/hierarchical approach," *IEEE Trans. Power Syst.*, vol. 16, no. 1, pp. 136–153, Feb. 2001.
- [26] J. W. Stahlhut, T. J. Browne, G. T. Heydt, and V. Vittal, "Latency Viewed as a Stochastic Process and its Impact on Wide Area Power System Control Signals," *IEEE Transactions on Power Systems*, vol. 23, no. 1, pp. 84-91, Feb. 2008.
- [27] T. Ohno, T. Yasuda, O. Takahashi, M. Kaminaga, and S. Imai, "Islanding Protection System based on Synchronized Phasor Measurements and its Operational Experiences," *Proceedings IEEE PES General Meeting*, pp. 1-5, 2008.
- [28] F. Galvin and C. Wells, "Detecting and Managing the Electrical Island Created by Hurricane Gustav," North American SynchroPhasor Initiative Success story. Available online: [http://www.naspi.org/stories/pilot\\_fundamental/entergy\\_hurricane\\_gustav.pdf](http://www.naspi.org/stories/pilot_fundamental/entergy_hurricane_gustav.pdf)
- [29] I. Slutsker, S. Mokhtari, L. Jaques, J. Provost, M. Perez, J. Sierra, F. Gonzalez, and J. Figueroa, "Implementation of phasor measurements in state estimator at Sevillana de Electricidad," in *IEEE Power Industry Computer Application Conference Proceedings*, May 1995, pp. 392–398.
- [30] M. Parashar, *et al.*, "Implementation of Phasor Measurements in SDG&E State Estimator," California Energy Commission, PIER Program Energy Commission - 500-02-04 MR053, Tech. Rep., 2008.
- [31] R. Avila-Rosales, M. J. Rice, J. Giri, L. Beard, and F. Galvan, "Recent Experience with a Hybrid SCADA/PMU On-line State Estimator," in *IEEE Power and Energy Society General Meeting*, July 2009.
- [32] B. Fardanesh, "Use of Phasor Measurements in a Commercial (or Industrial) State Estimator," EPRI, Palo Alto, California, Final Report1011002, July 2004.
- [33] A. Ghassemian and B. Fardanesh, "Phasor assisted state estimation for NYS transmission system - Implementation & Testing," in *IEEE PES Power Systems Conference and Exposition*, March 2009.
- [34] D. Atanackovic, J. Clapauch, G. Dwernychuk, J. Gurney, and H. Lee, "First steps to wide area control," *IEEE Power and Energy Magazine*, vol. 6, no. 1, pp. 61–68, January-February 2008.
- [35] L. Vanfretti, J. H. Chow, S. Sarawgi, D. Ellis, and B. Fardanesh, "A Framework for Estimation of Power Systems Based on Synchronized Phasor Measurement Data," in *IEEE PES General Meeting*, July 2009, pp. 1-6.
- [36] L. Vanfretti, J. H. Chow, S. Sarawgi, and B. Fardanesh, "A Phasor-Data Based State Estimator Incorporating Phase Bias Correction," to appear in *IEEE Transactions on Power Systems*, 2011.
- [37] J. H. Chow, L. Vanfretti, et al, "Preliminary Synchronized Phasor Data Analysis of Disturbance Events in the US Eastern Interconnection," *IEEE PES Power System Conference and Exposition (PSCE) 2009*, pp. 1-8, March 2009.

- [38] K. Vu, M. M. Begovic, D. Novosel, and M. M. Saha, "Use of local measurements to estimate voltage stability margin," *IEEE Transactions on Power Systems*, vol. 14, no. 3, pp. 1029–1035, Aug. 1999.
- [39] D.E. Julian, R.P. Schulz, K.T. Vu., W.H. Quaintance, N.B. Bhatt, D. Novosel, "Quantifying proximity to voltage collapse using the Voltage Instability Predictor (VIP)," *IEEE PES Summer Meeting 2000*, vol.2, no., pp.931-936 vol. 2, 2000.
- [40] G. Verbic, and F. Gubina, "A new concept of voltage-collapse protection based on local phasors," *IEEE Transactions on Power Delivery*, vol. 19, no .2, pp. 576-581, April 2004.
- [41] I. Smon, G. Verbic, and F. Gubina, "Local voltage-stability index using Tellegen's theorem," *IEEE Transactions on Power Systems*, vol. 21, no. 3, pp. 1267–1275, Aug. 2006.
- [42] D.N. Kosterev, C.W. Taylor, W.A. Mittelstadt, "Model validation for the August 10, 1996 WSCC system outage," *IEEE Transactions on Power Systems*, vol. 14, no. 3, pp. 967-979, Aug 1999.
- [43] J.H. Chow, M.T. Glinkowski, R.J. Murphy, T.W. Cease, and N. Kosaka, "Generator and exciter parameter estimation of Fort Patrick Henry Hydro Unit 1," *IEEE Transactions on Energy Conversion*, vol. 14, no. 4, pp. 923-929, Dec 1999.
- [44] "IEEE Std 421.5 - 2005 IEEE Recommended Practice for Excitation System Models for Power System Stability Studies," *IEEE Std 421.5-2005 (Revision of IEEE Std 421.5-1992)*, pp. 1-85, 2006.
- [45] "IEEE Guide: Test Procedures for Synchronous Machines," *IEEE Std 115-1995*, 12 Dec 1995.
- [46] S.M. Benchluch and J.H. Chow, "A trajectory sensitivity method for the identification of nonlinear excitation system models," *IEEE Transactions on Energy Conversion*, vol. 8, no. 2, pp. 159-164, June 1993.
- [47] V. Madani, "PMU Placement Consideration Factors," June 9, 2010.  
[http://www.naspi.org/meetings/workgroup/2010\\_june/presentations/session\\_04/madani\\_pacific\\_gas\\_pmu\\_placement\\_20100609.pdf](http://www.naspi.org/meetings/workgroup/2010_june/presentations/session_04/madani_pacific_gas_pmu_placement_20100609.pdf) .
- [48] M. Patel, et al., "Real-Time Application of Synchrophasors for Improving Reliability," NERC Report, Oct. 2010. Available online at <http://www.naspi.org/resources/resources.stm>.