Designing Model-Free Time Derivatives in the Frequency Domain for Ambient PMU Data Applications

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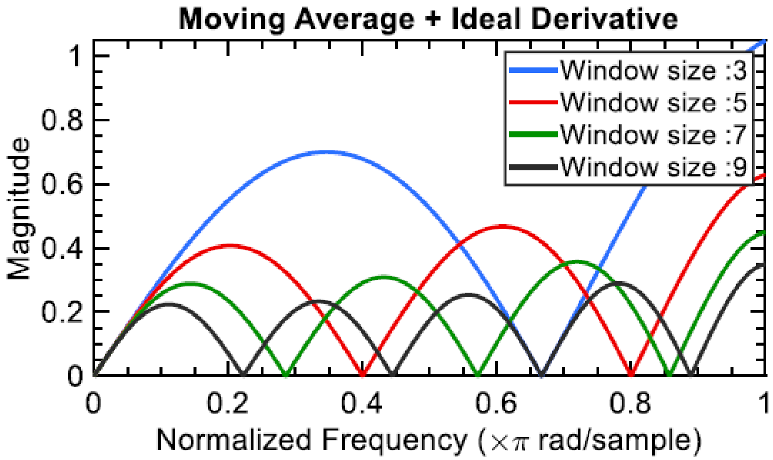
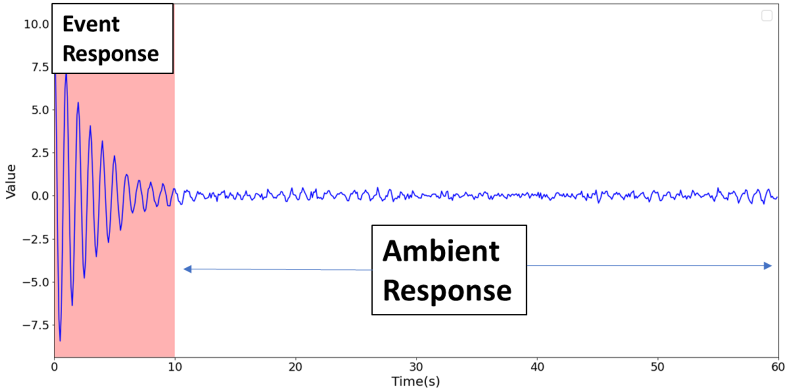
Dominion Energy

Background

- Model-free derivatives are essential to synchrophasor applications such as frequency estimation, inertia monitoring, sensitivity analysis, etc
  - Each application is typically restricted to a given time scale, for e.g. Thevenin equivalent for steady state/longer time scale
- Ideal derivative + arbitrary level of smoothing based on visual inspection
- Difficult to do above in ambient conditions
  - Spectral analysis can yield valuable insights into system dynamics
- Introducing a framework for designing high order derivatives in frequency domain for ambient data applications
  - Capable of accounting for time scales of interest




Event Response

Ambient Response

Derivative Design using Maxflat

- Shaping transfer function taylor terms to match desirable gain
- Ideal n^{th}order derivative in frequency domain

$$\mathcal{F}\left(y^{(n)}(t)\right) = (j\omega)^n Y(\omega)$$

- Designed as a linear filter

$$Y_{filt}(\omega) = \left(\sum_i G_i(\omega^*)(j\omega - j\omega^*)^i\right) Y(\omega)$$

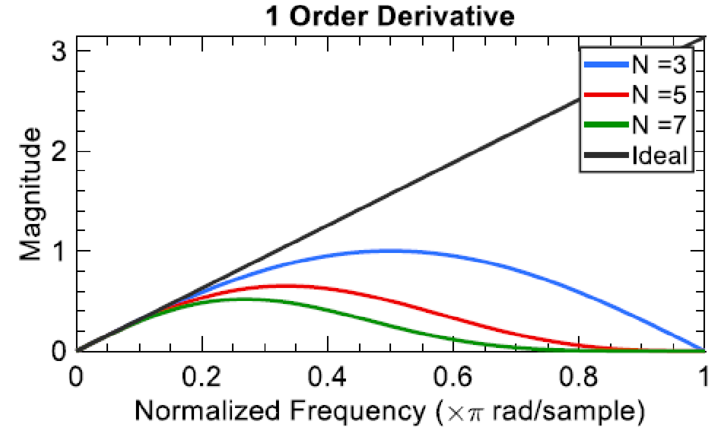
$$G_i(\omega^*) = \left(\sum_{k=1:N} \frac{b_k(-k\Delta t)^i e^{-jk\omega^*\Delta t}}{i!}\right)$$

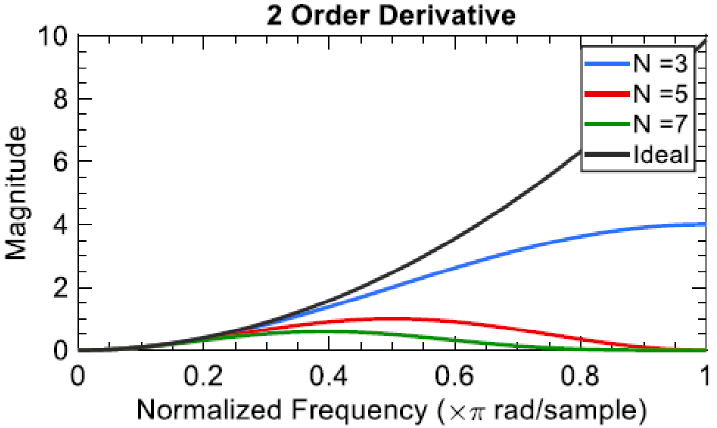
- Design specifications
  - Matches ideal derivative at low frequencies

$$G_i(0) = 0, \forall\, 0 \le i \le n-1$$

  - Suppresses high frequencies (noise + faster irrelevant dynamics)

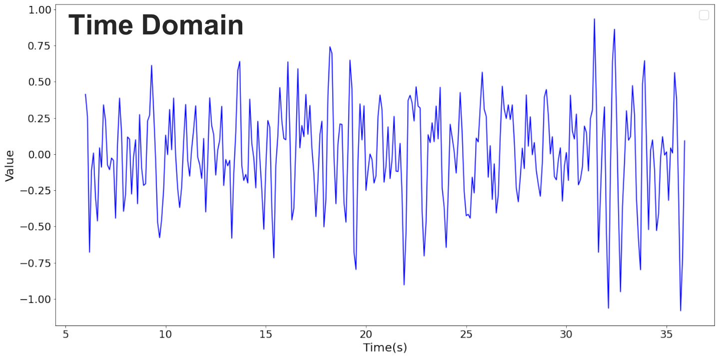
$$G_i(\pi f_s) = 0, \forall\, 0 \le i \le N-(n+2)$$

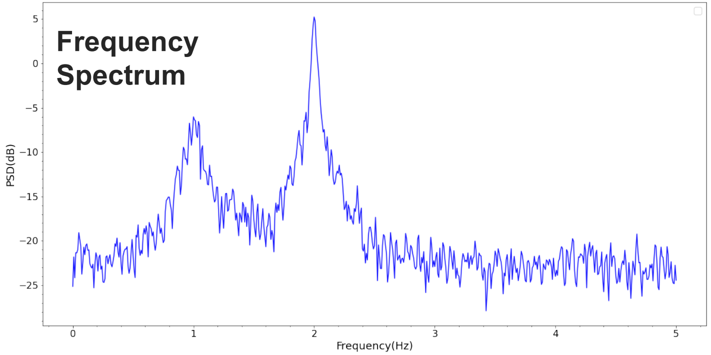




Choosing Appropriate Window Size N

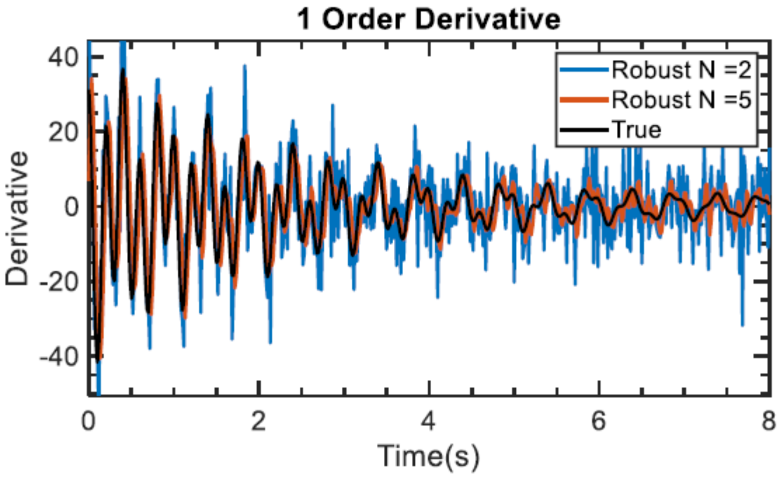
- Increased higher frequency suppression with N
  - Should be chosen to not result in loss of relevant system behavior
- Power system in ambient conditions can be modeled as stochastic linear system
- **Output Frequency Spectrum** - $S_{yy}(\omega) = \sum_i \left[\frac{A_i}{j\omega - \lambda_i} + \frac{A_i^*}{j\omega + \lambda_i^*}\right]$
  - λ_i characterize the time scales of the underlying system dynamics
  - Can be estimated from measurements only => full knowledge of what frequencies can be suppressed in derivative

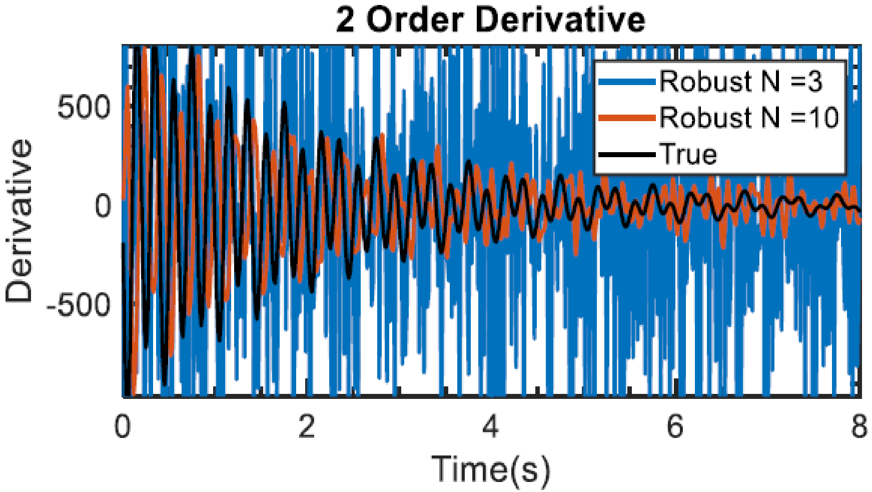




Results – Synthetic Ringdown Signal

$$y(t) = e^{-0.5t}\sin(2\pi 5t) + e^{-0.2t}\cos(2\pi 2t) + \epsilon(t)$$

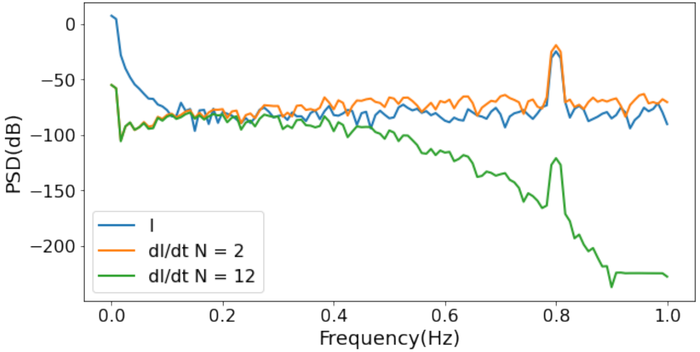


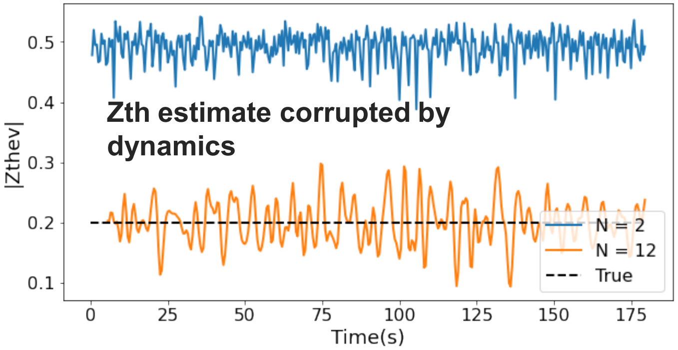


Results – Thevenin Equivalent

- Thevenin equivalent is generally used to approximate the behavior of the rest of the system to local changes in load using a voltage source V_{th} behind an impedance Z_{th}
  - For steady state voltage stability analysis.
  - Z_{th} satisifies $-\Delta \boldsymbol{I}_{\boldsymbol{ep}}(\boldsymbol{t}) Z_{th} = \Delta \boldsymbol{V}_{\boldsymbol{ep}}(\boldsymbol{t})$ where, $(V_{ep}(t), I_{ep}(t))$ is the trajectory of operating point/equilibrium
- Measured data contains both equilibrium trajectory as well as dynamics around it $\big(V(t), I(t)\big) = \big(V_{dyn}(t) + V_{ep}(t), I_{dyn}(t) + I_{ep}(t)\big)$
  - Equilibrium trajectory is significantly slower than dynamics, mostly separable in frequency domain...can use robust derivative to estimate $\Delta \boldsymbol{I}_{\boldsymbol{ep}}(\boldsymbol{t}), \Delta \boldsymbol{V}_{\boldsymbol{ep}}(\boldsymbol{t})$!

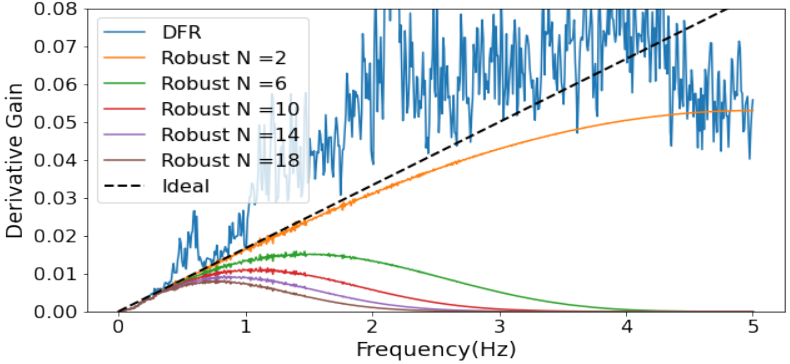
Load current with slow ramping and 0.8 Hz mode



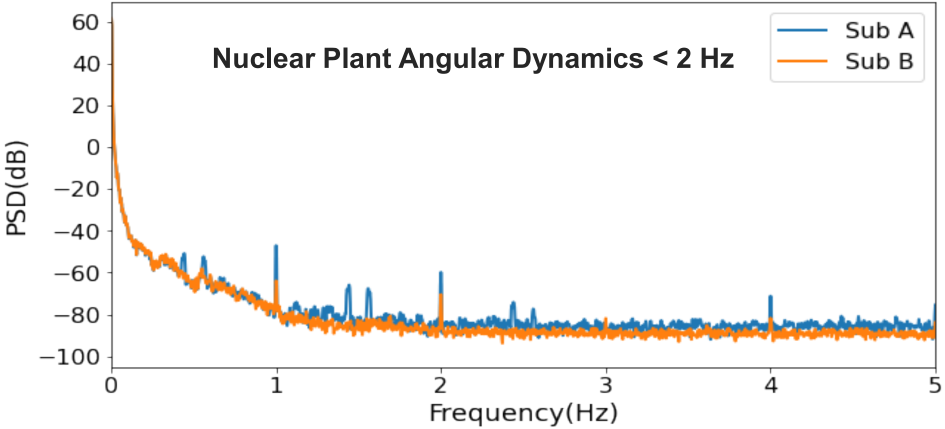

Zth estimate corrupted by dynamics

Results – Frequency Estimation

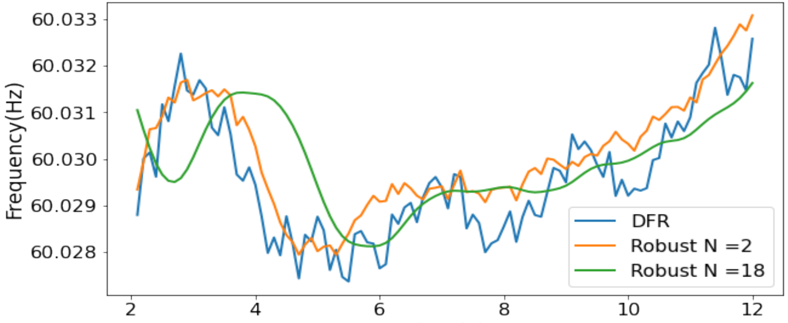
- Loosely represents derivative of angle $f = \frac{\dot{\theta}}{2\pi}$
- Estimation algorithms differ among vendors
  - Unable to compare results



Original DFR Freq Estimate Retains High Frequencies



Nuclear Plant Angular Dynamics < 2 Hz



Window Size 18 Retains Only Appropriate Dynamics with Sufficient Smoothness

Conclusions/Recommendations

- Derivative estimation framework that accounts for time scale of interest of underlying system dynamics
- Future work will explore using this for state estimation and control design applications