Iterative-Interpolated DFT for Synchrophasor Estimation: a Single Algorithm for P and M-class compliant PMUs

École Polytechnique Fédérale de Lausanne (EPFL)
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Outline

- Motivations
- Problem statement
- The Interpolated DFT for $\cos^{\alpha}$ windows
- The iterative Interpolated DFT
- Performance assessment
- Computational complexity

Motivations

P and M-class compliant PMU (IEEE Std. C37.118)

- The IEEE Std. C37.118 defines two performance classes to which PMUs must comply with:

↙ ↘

P-class	**M-class**
Protection → Latency	Measurement → Accuracy

- *What if we had a single PMU capable of satisfying both the P and M-class PMU requirements at the same time?*

↓

  - **Lower cost**: the same PMU simultaneously supplies monitoring and protection functionalities
  - **Higher measurement reliability**: protection and control applications are not degraded by disturbances (interharmonics)

Motivations

The Out-of-band Interference (OOBI) test

		P-class	M-class
Static conditions	Signal Frequency	✓	✓
	Harmonic Distortion	✓	✓
	Out-Of-Band Interference	✗	✓
Dynamic conditions	Measurement Bandwidth	✓	✓
	Frequency Ramp	✓	✓
	Amplitude Phase Step	✓	✓

PMU capability to reject **interharmonics** close to the main tone:

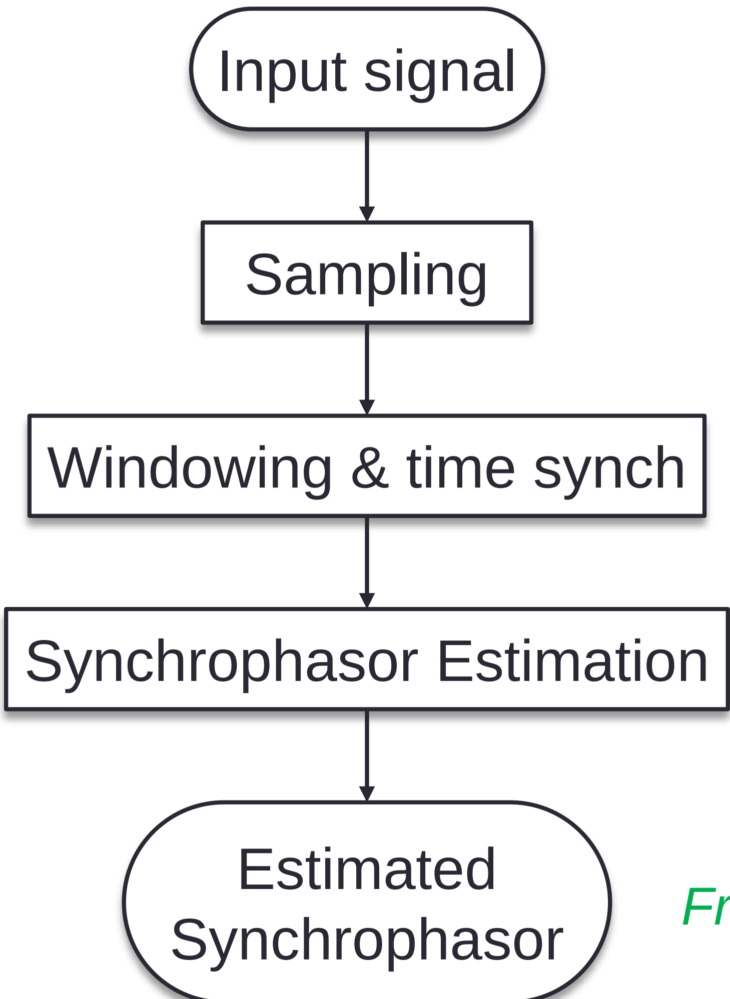
- $A_i = 10\% \cdot A_0$
- $f_i \in [10\text{ Hz}, \quad f_N - F_R/2] \cup [f_n + F_R/2, \quad 2 \cdot fn]$
- $f_0/f_\text{i} \notin \mathbb{N}$

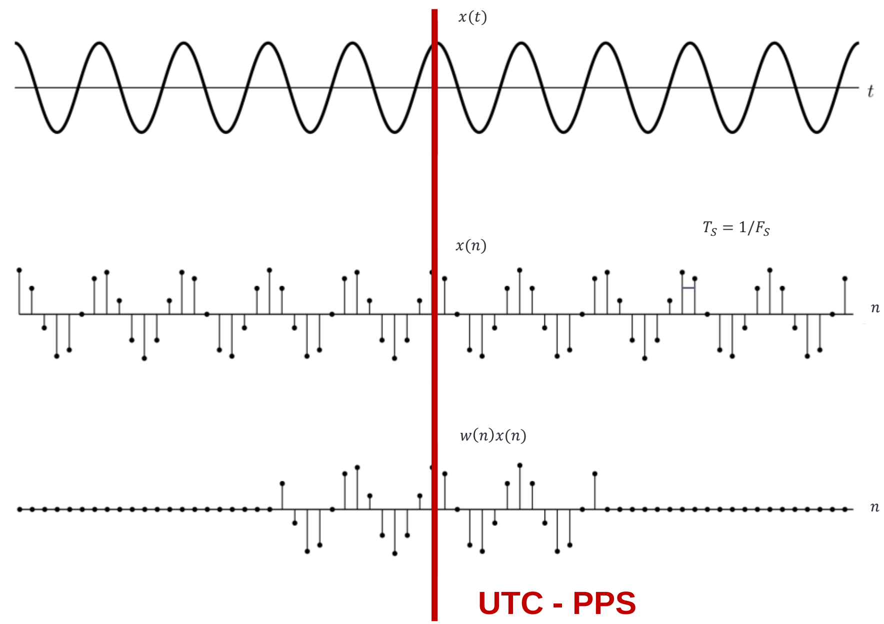
- $f_N = 50$ Hz (nominal frequency)
- $F_R = 50$ fps (PMU reporting rate)

→ $f_0 = \quad [47.5, 50, 52.5]$ Hz
$f_i = \quad [10{:}\,25 \; 75{:}\,100]$ Hz

Problem statement

DFT-based Synchrophasor Estimation (SE) Algorithms





$x(n) \rightarrow X(k) \rightarrow$ **?**

Frequency f_0, *Amplitude* A_0 *and Phase* φ_0

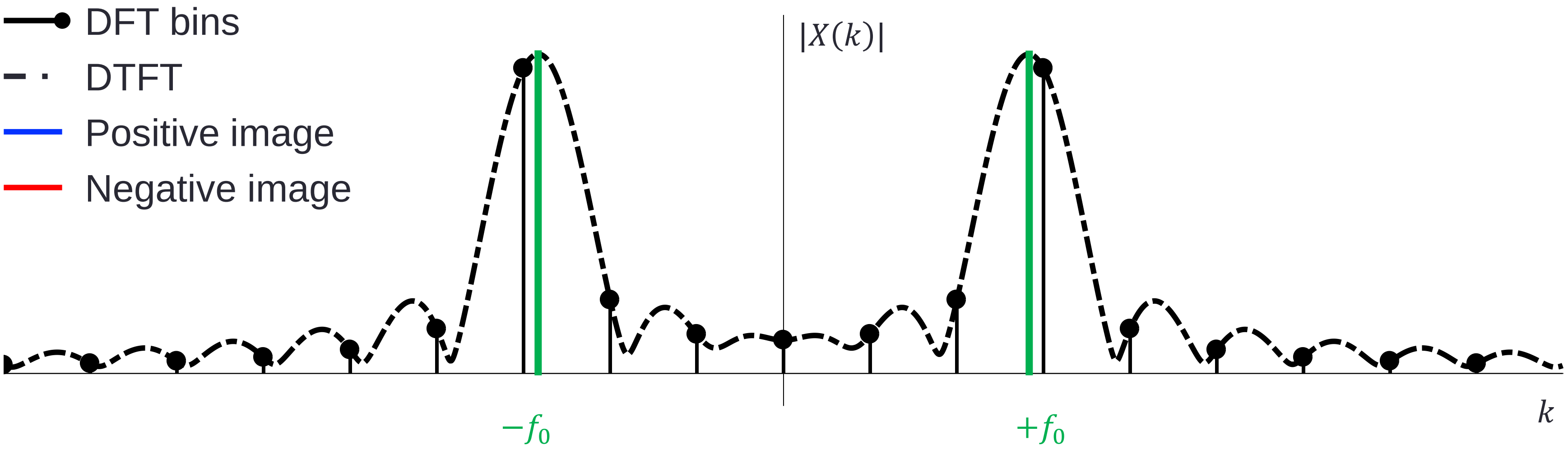
Problem statement

The Signal Frequency test

$$x(t) = A_0(t)\cos(2\pi f_0(t)t + \varphi_0)$$

$$\Downarrow$$

$$X(k) = X_0^+(k) + X_0^-(k)$$



Find the frequency f_0, amplitude A_0 and phase φ_0 of the ***fundamental tone*** *of a signal*

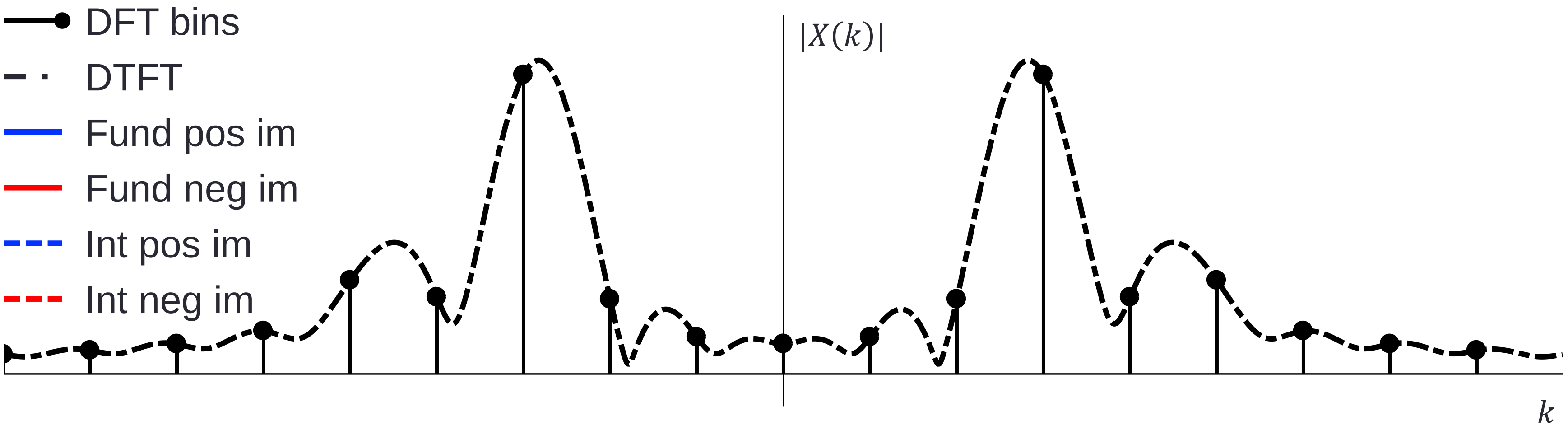
Problem statement

The Out-of-band Interference (OOBI) test

$$x(t) = A_0(t)\cos(2\pi f_0(t)t + \varphi_0) + A_i(t)\cos(2\pi f_i(t)t + \varphi_i)$$

$f_0/f_\mathrm{i} \notin \mathbb{N}$

$$X(k) = X_0^+(k) + X_0^-(k) + X_i^+(k) + X_i^-(k)$$



Find the frequency f_0, amplitude A_0 and phase φ_0 of the ***fundamental tone*** *of a signal characterized by two tones, a fundamental and an interharominc tone, both of unknown frequency amplitude and phase*

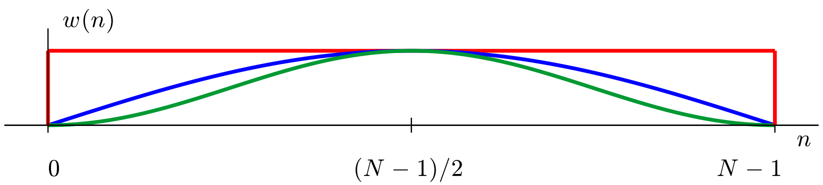
The Interpolated DFT (IpDFT)

IpDFT problem solution for $\cos^\alpha$ window functions

The IpDFT is a technique to extract the parameters f_0, A_0 and φ_0 of a sinusoidal waveform by interpolating the highest DFT bins of the signal spectrum. It mitigates the effects of **incoherent sampling** ($f_0/\Delta f \notin \mathbb{N}$):

- Applying special windowing functions ➔ reduce **spectral leakage**

$$w_\alpha(n) = \sin^\alpha\left(\frac{\pi n}{N}\right)$$



α = 0 ➔ Rectangular

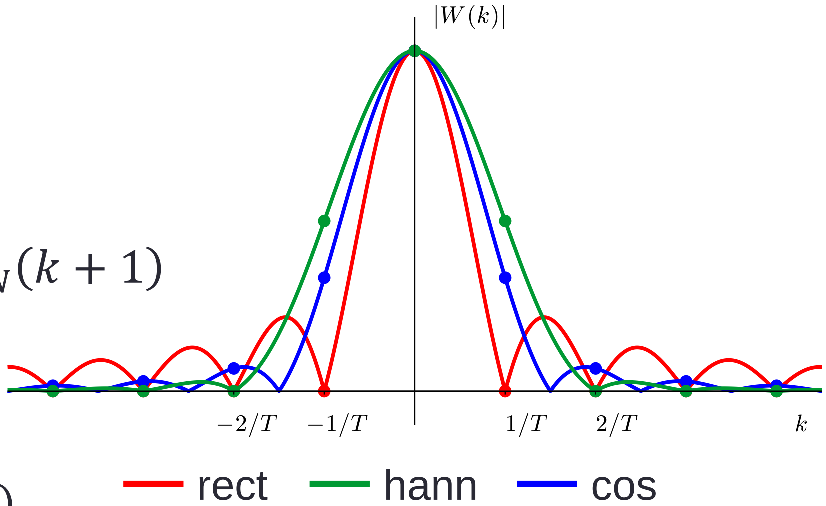
$$D_N(k) = e^{-j\pi k(N-1)/N}\frac{\sin(\pi k)}{\sin(\pi k/N)}$$

α = 2 ➔ Hanning (hann)

$$W_H(k) =$$
$$-0.25 \cdot D_N(k-1) + 0.5 \cdot D_N(k) - 0.25 \cdot D_N(k+1)$$

α = 1 ➔ Cosine (cos)

$$W_C(k) \ =$$
$$-0.5 \cdot j \cdot D_N(k-0.5) + 0.5 \cdot j \cdot D_N(k+0.5)$$



The Interpolated DFT (IpDFT)

IpDFT problem solution for $\cos^\alpha$ window functions

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- Interpolating the highest DFT bins ➔ minimize **spectral sampling**

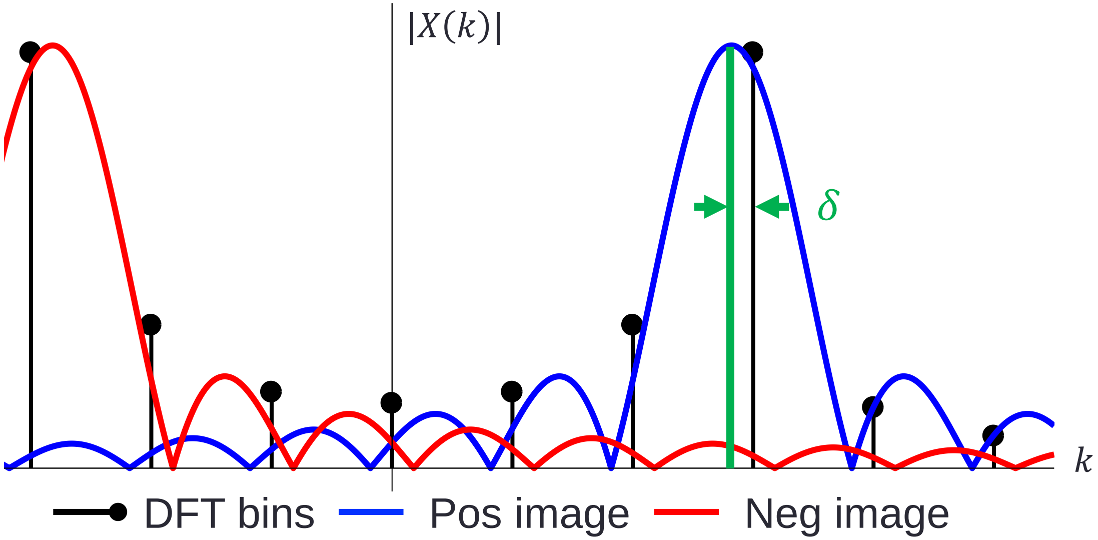
$$\delta = a \cdot \varepsilon \frac{|X(k_m+\varepsilon)| - |X(k_m-\varepsilon)|}{|X(k_m-\varepsilon)| + 2|X(k_m)| + |X(k_m+\varepsilon)|},$$ a = 1.5 cos, a = 2 hann

$$f_0 = (k_m + \delta)\Delta f$$

$$\varphi_0 = \angle X(k_m) - \pi\delta$$

$$A_{0C} = 4 \cdot |X(k_m)| \left|\frac{\delta^2 - 0.25}{\cos(\pi\delta)}\right|$$

$$A_{0H} = |X(k_m)| \left|\frac{\pi\delta}{\sin(\pi\delta)}\right| |\delta^2 - 1|$$



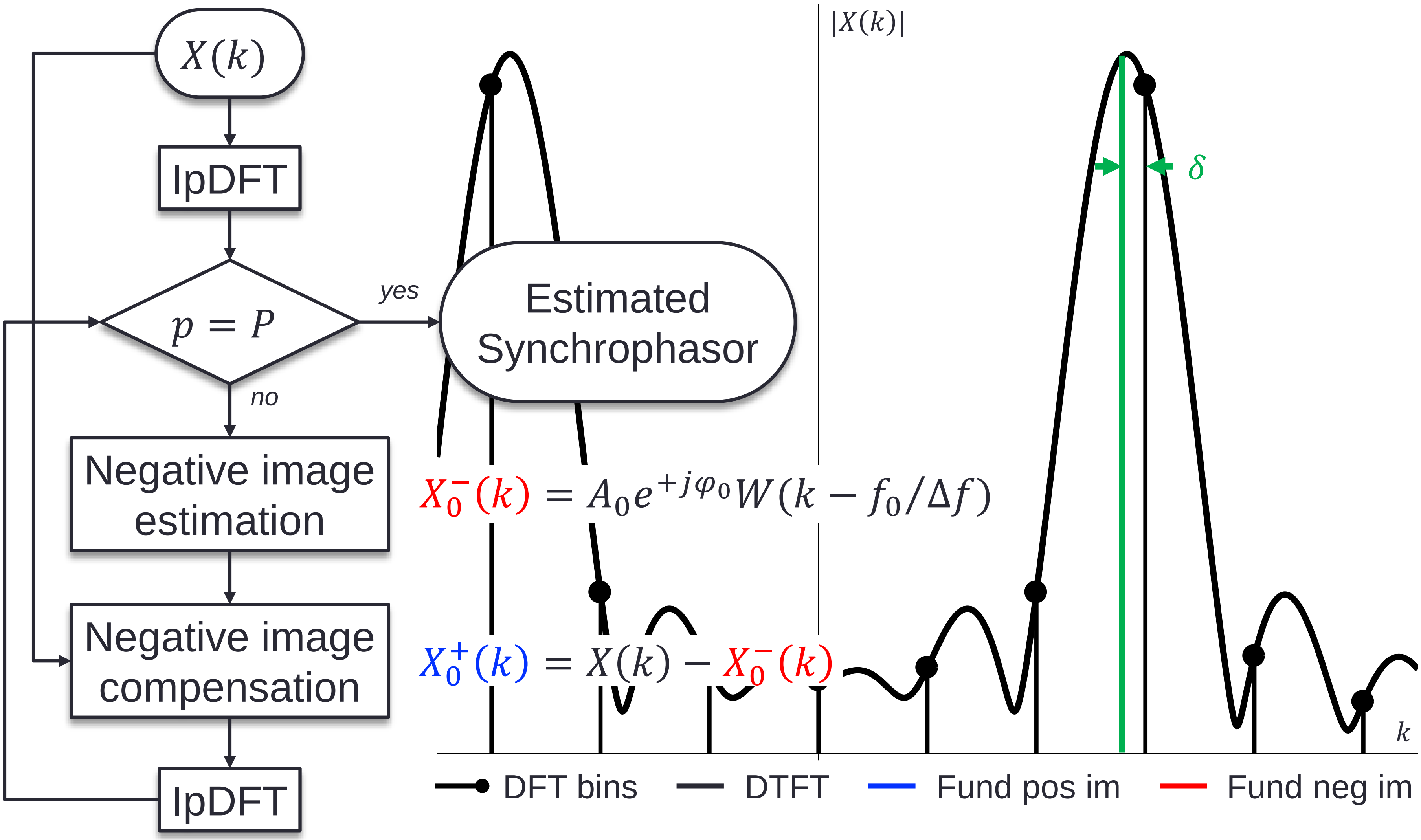
The Interpolated DFT (IpDFT)

IpDFT weaknesses

Assumptions behind the IpDFT	Possible solutions
The input signal is characterized by time-invariant parameters	Window lengths containing few periods of a signal at the rated power system frequency
The sampling rate is higher than the highest signal's spectral component	Sampling rate F_S in the order of tens of kHz
The DFT bins used to perform the interpolation are only generated by the positive image of the tone under analysis	▪ Enhanced IpDFT (e-IpDFT) ➔ Iterative compensation of the spectral interference produced by the main tone negative image ▪ **Iterative IpDFT (i-IpDFT) ➔** Iterative compensation of the spectral interference produced by the nearby tones

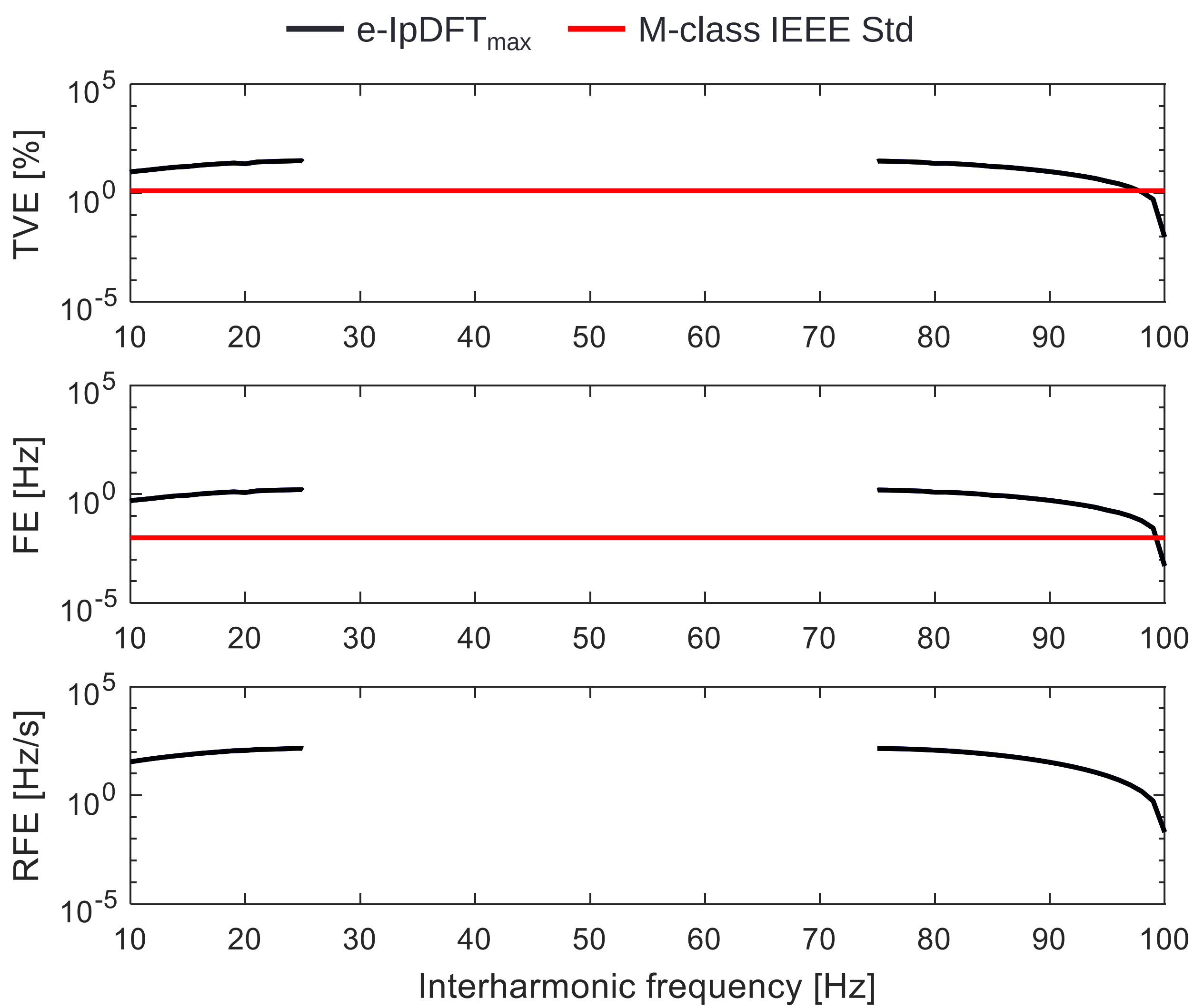
The enhanced-IpDFT (e-IpDFT)

Algorithm formulation



The enhanced-IpDFT (e-IpDFT)

The Out-Of-Band Interference test



Max Errors:

TVE

IEEE Std = 1.3%

e-IpDFT = 31%

FE

IEEE Std = 10 mHz

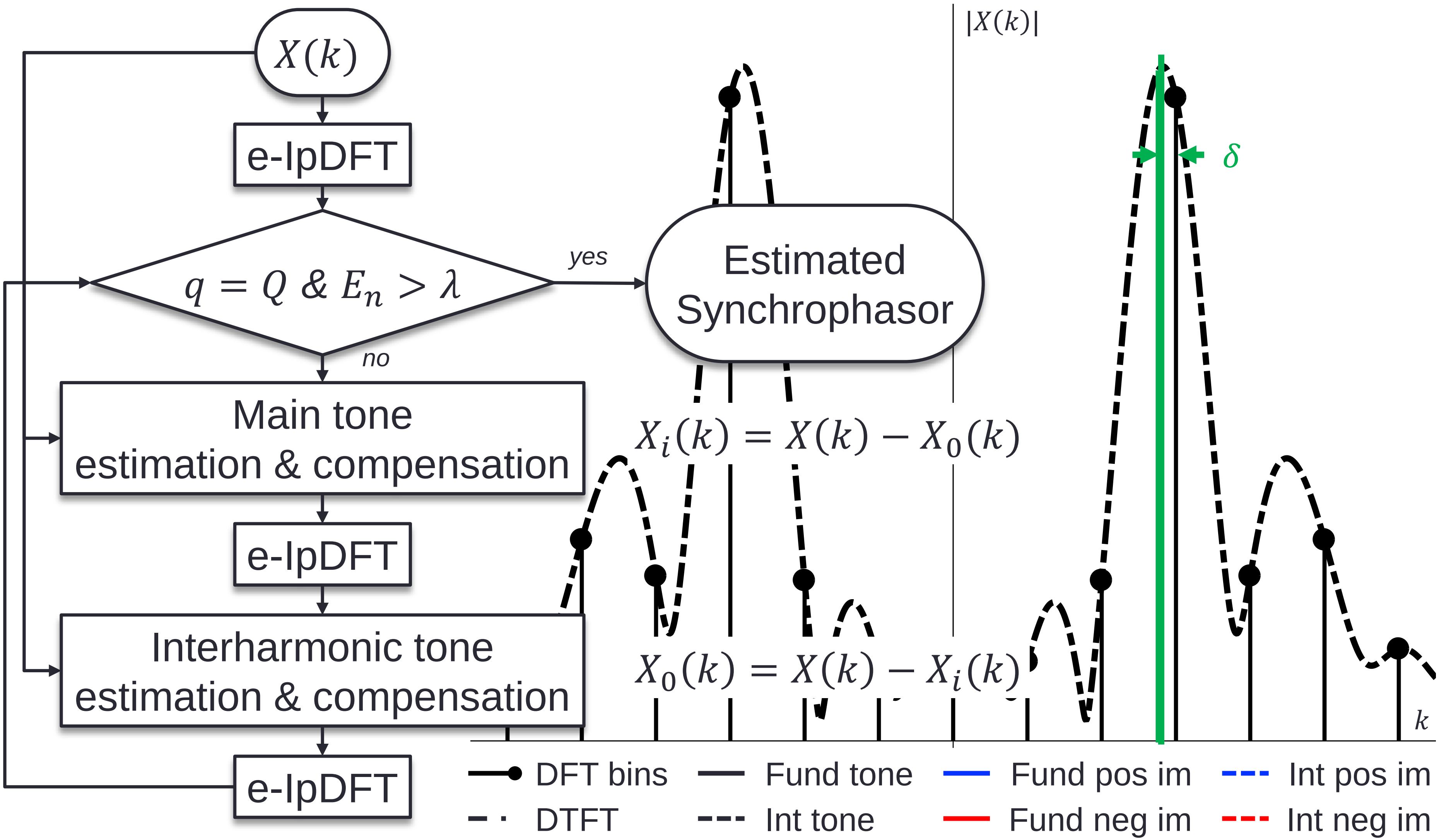
e-IpDFT = 1.6 Hz

RFE

e-IpDFT = 145 Hz/s

The iterative-IpDFT (i-IpDFT)

Algorithm formulation



The iterative-IpDFT (i-IpDFT)

On the tuning of the number of iterations P and Q

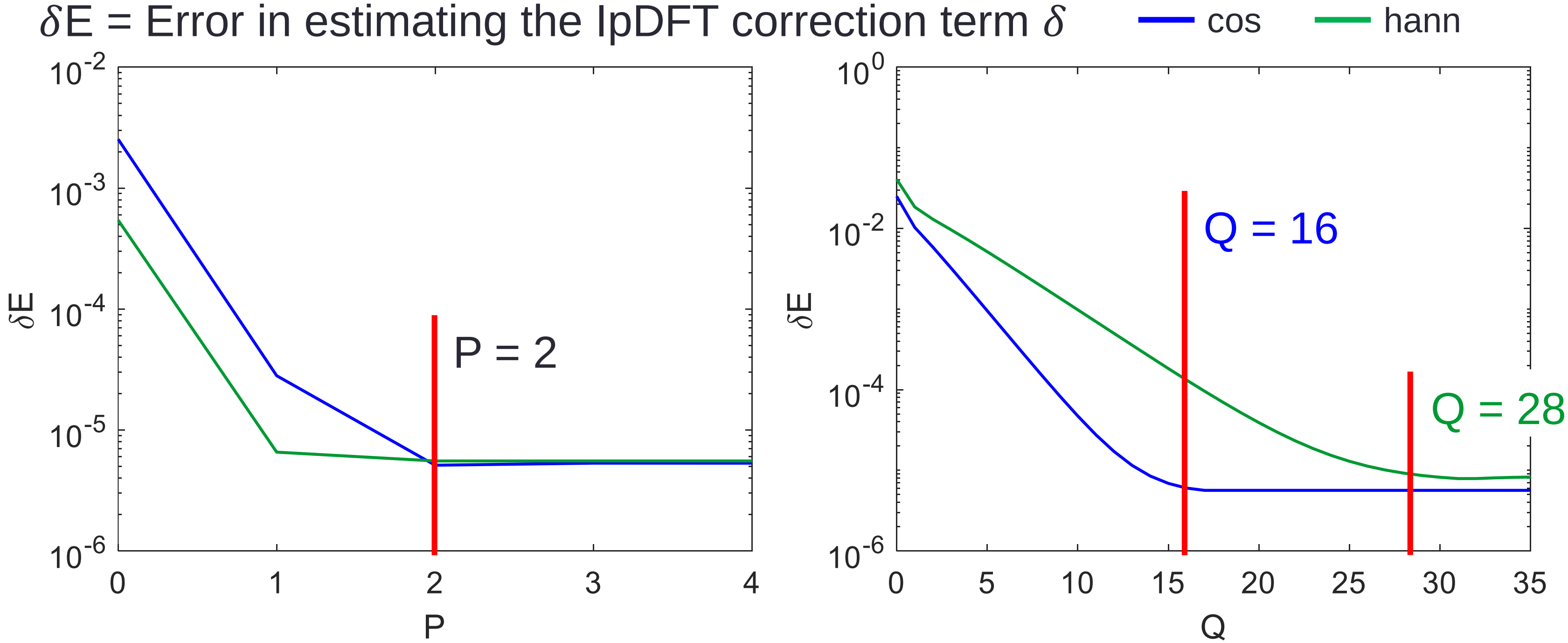
P ➔ # iterations compensation negative image of the tone

Q ➔ # iterations overall procedure

e-IpDFT ➔ f_0 = 47.5 Hz

i-IpDFT ➔ f_0 = 47.5 Hz, f_i = 20 Hz

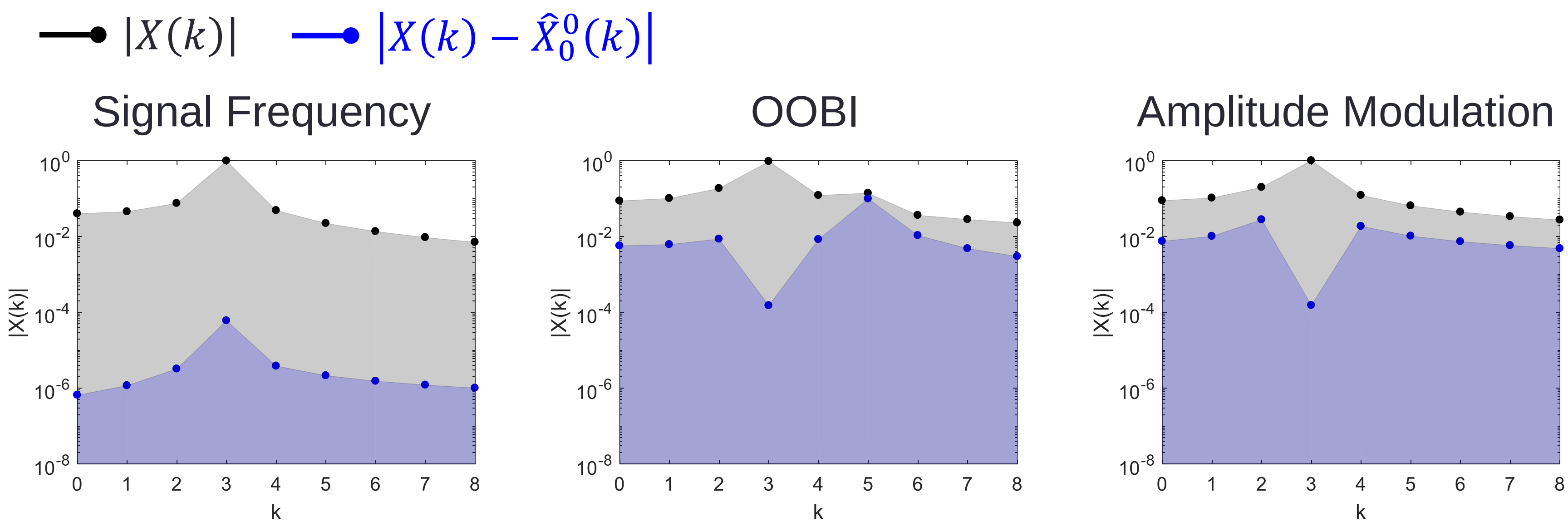
δE = Error in estimating the IpDFT correction term δ



The iterative-IpDFT (i-IpDFT)

On the tuning of the threshold λ

$$E_n = \frac{E\left[X(k) - \hat{X}_0^0(k)\right]}{E\left[X(k)\right]} = \frac{\sum_{k=0}^{K}\left|X(k) - \hat{X}_0^0(k)\right|^2}{\sum_{k=0}^{K}|X(k)|^2} > \lambda$$

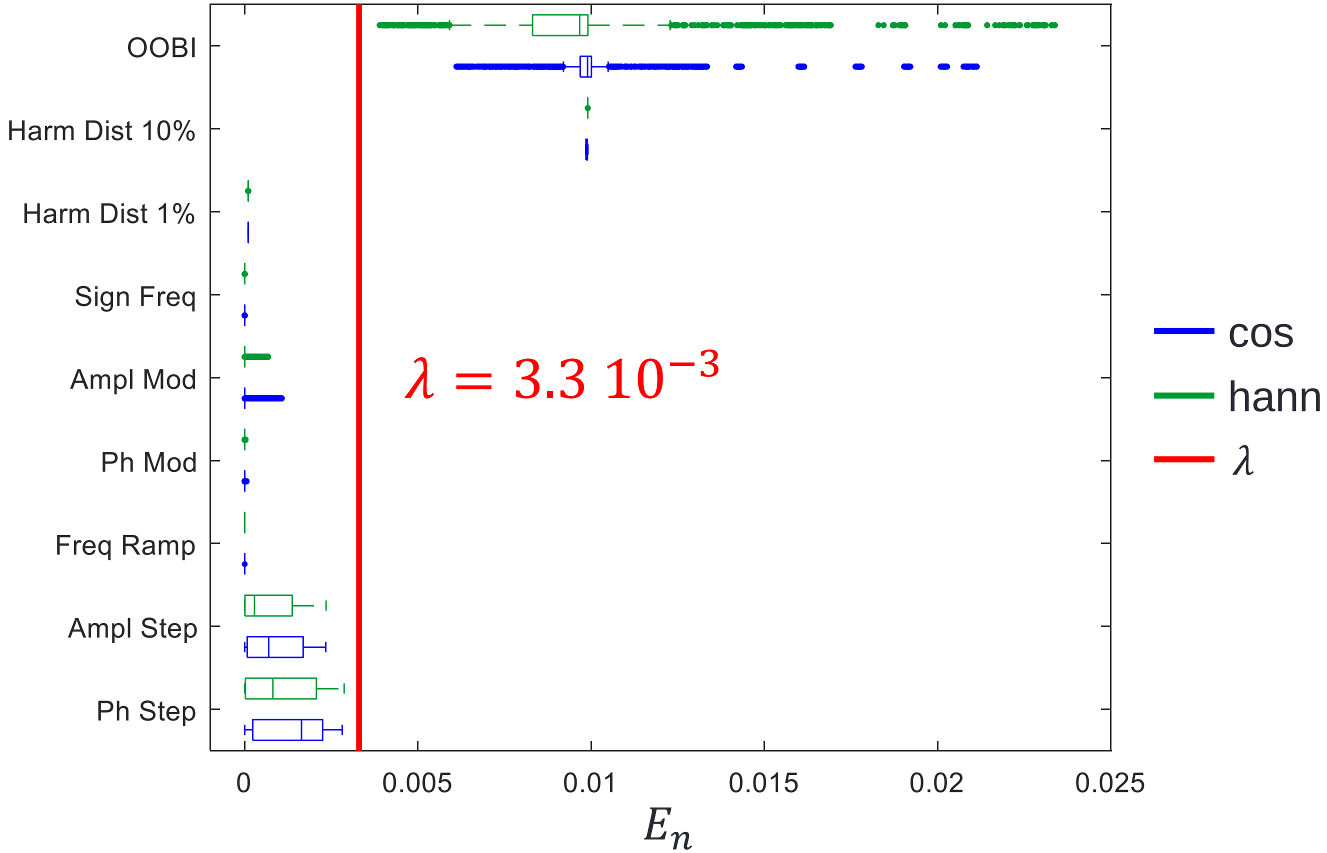


E_n low | **E_n high** | E_n low

The iterative-IpDFT (i-IpDFT)

On the tuning of the threshold λ

$$E_n = \frac{E\left[X(k) - \hat{X}_0^0(k)\right]}{E\left[X(k)\right]} = \frac{\sum_{k=0}^{K}\left|X(k) - \hat{X}_0^0(k)\right|^2}{\sum_{k=0}^{K}\left|X(k)\right|^2} > \lambda$$



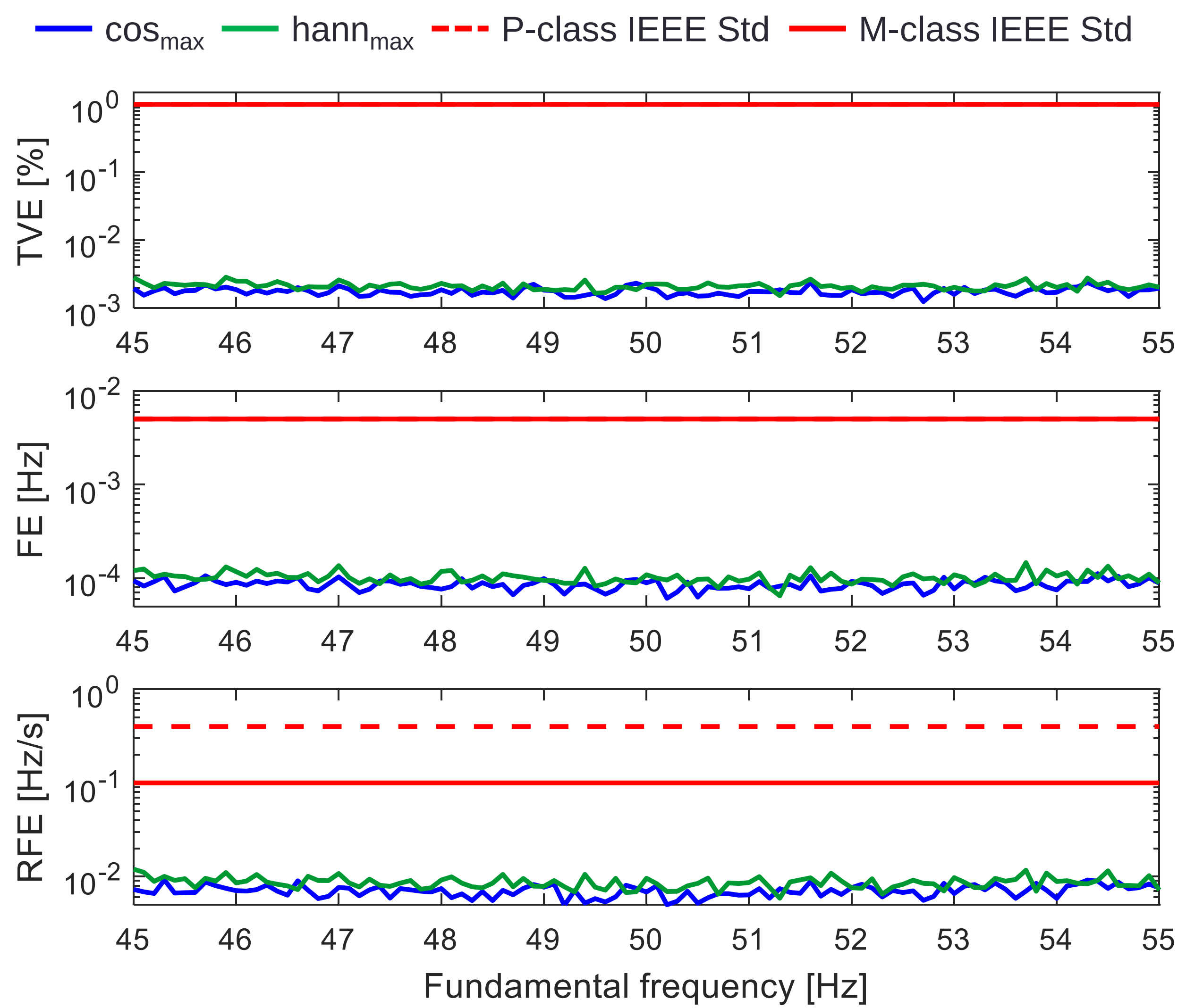
Performance Assessment

i-IpDFT Parameters

Parameters	Variable	Value
Nominal system frequency	f_0	50 Hz
Window length	T	$3 \cdot T_0$ = 60 ms
Sampling rate	F_S	50 kHz
PMU reporting rate	F_R	50 fps
# DFT bins	K	11
IpDFT interpolation points		3-points
# iterations	P	2
# iterations	Q	16 (cos) 28 (Hann)
Noise		80 dB

Performance Assessment

Static conditions – Signal Frequency



Max Errors:

TVE

P&M = 1%

cos = 0.002%

hann = 0.003%

FE

P&M = 5 mHz

cos = 0.1 mHz

hann = 0.1 mHz

RFE

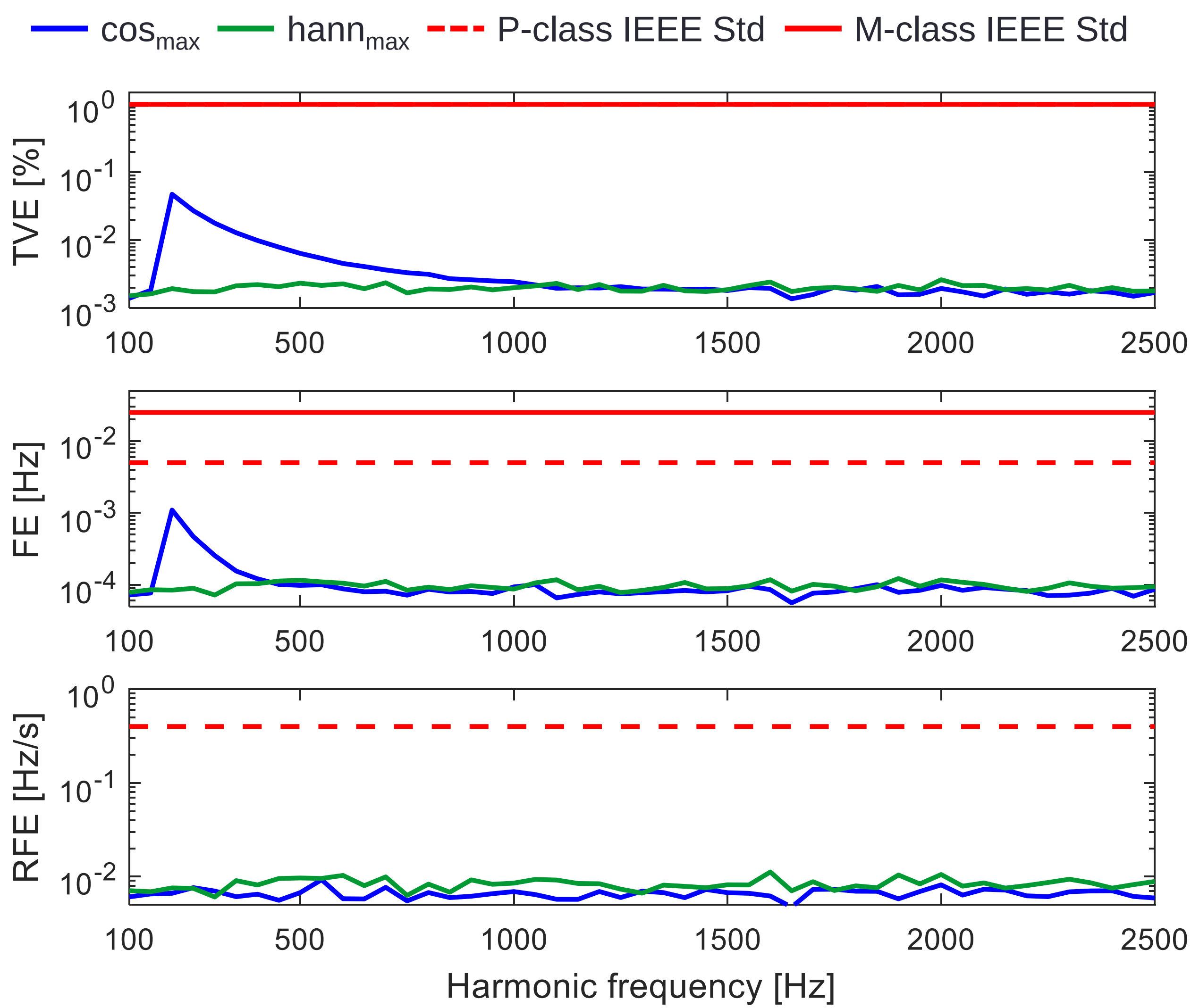
P = 0.4, M = 0.1 Hz/s

cos = 0.01 Hz/s

hann = 0.01 Hz/s

Performance Assessment

Static conditions – Harmonic Distortion



Max Errors:

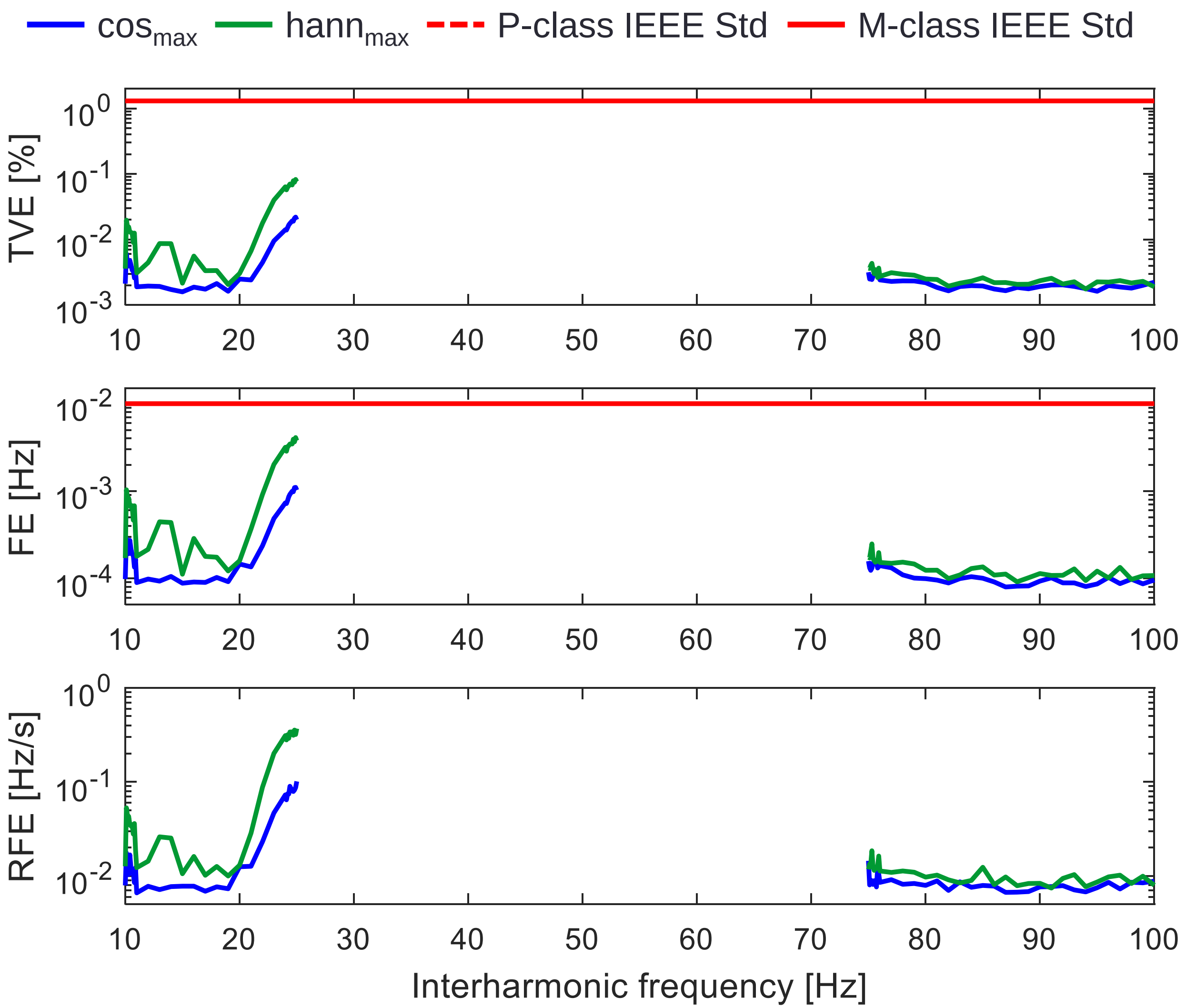
TVE
P&M = 1%
cos = 0.05%
hann = 0.003%

FE
P = 5, M = 25 mHz
cos = 1.1 mHz
hann = 0.1 mHz

RFE
P = 0.4 Hz/s
cos = 0.01 Hz/s
hann = 0.01 Hz/s

Performance Assessment

Static conditions – OOBI



Max Errors:

TVE

M = 1.3%

cos = 0.02%

hann = 0.08%

FE

M = 10 mHz

cos = 1.1 mHz

hann = 4.1 mHz

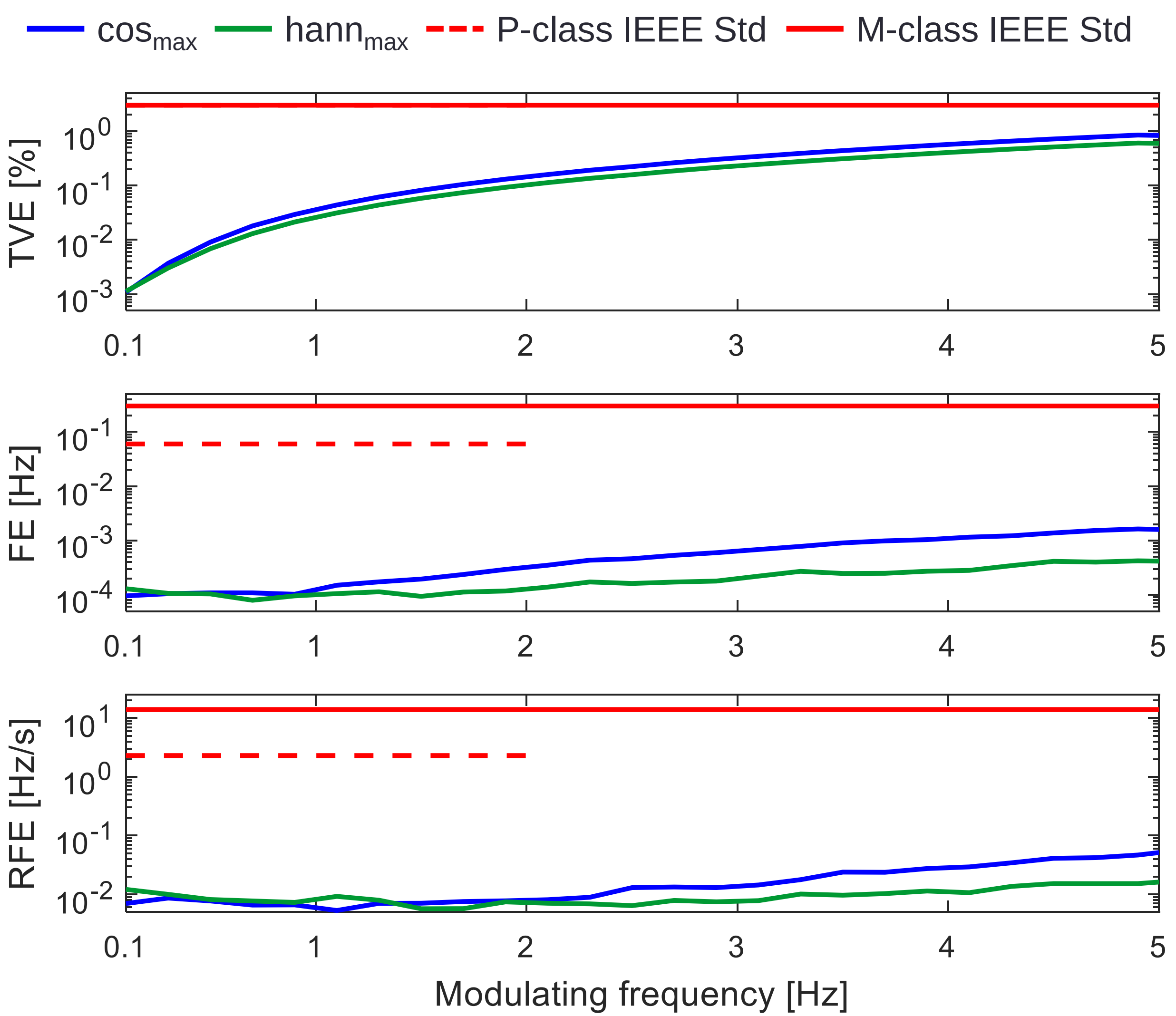
RFE

cos = 0.1 Hz/s

hann = 0.4 Hz/s

Performance Assessment

Dynamic conditions – Amplitude Modulation



Max Errors:

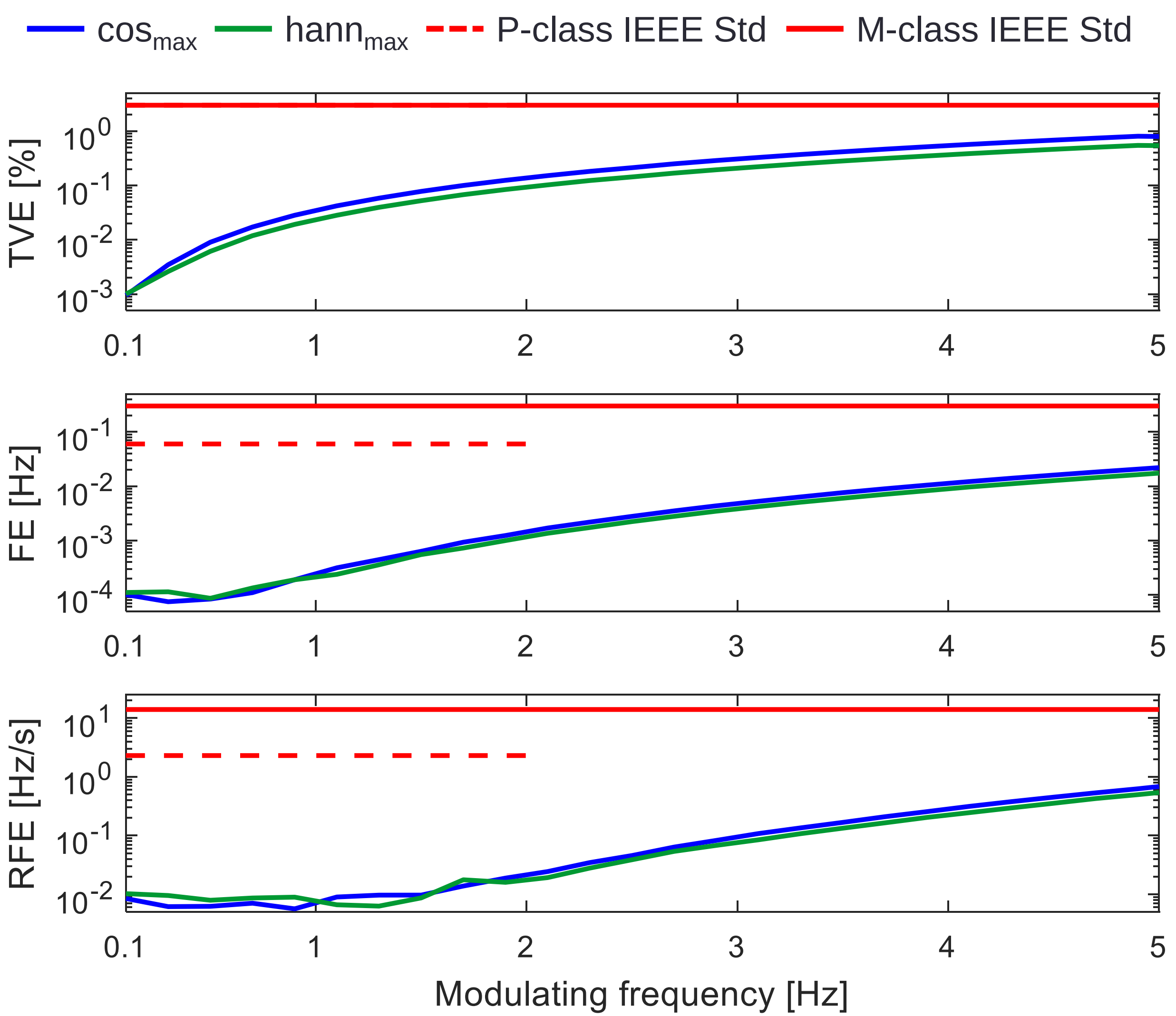
TVE
P&M = 3%
cos = 0.8%
hann = 0.6%

FE
P = 60, M = 300 mHz
cos = 1.6 mHz
hann = 0.4 mHz

RFE
P = 2.3, M = 14 Hz/s
cos = 0.05 Hz/s
hann = 0.02 Hz/s

Performance Assessment

Dynamic conditions – Phase Modulation



Max Errors:

TVE

P&M = 3%

cos = 0.8%

hann = 0.5%

FE

P = 60, M = 300 mHz

cos = 22 mHz

hann = 17 mHz

RFE

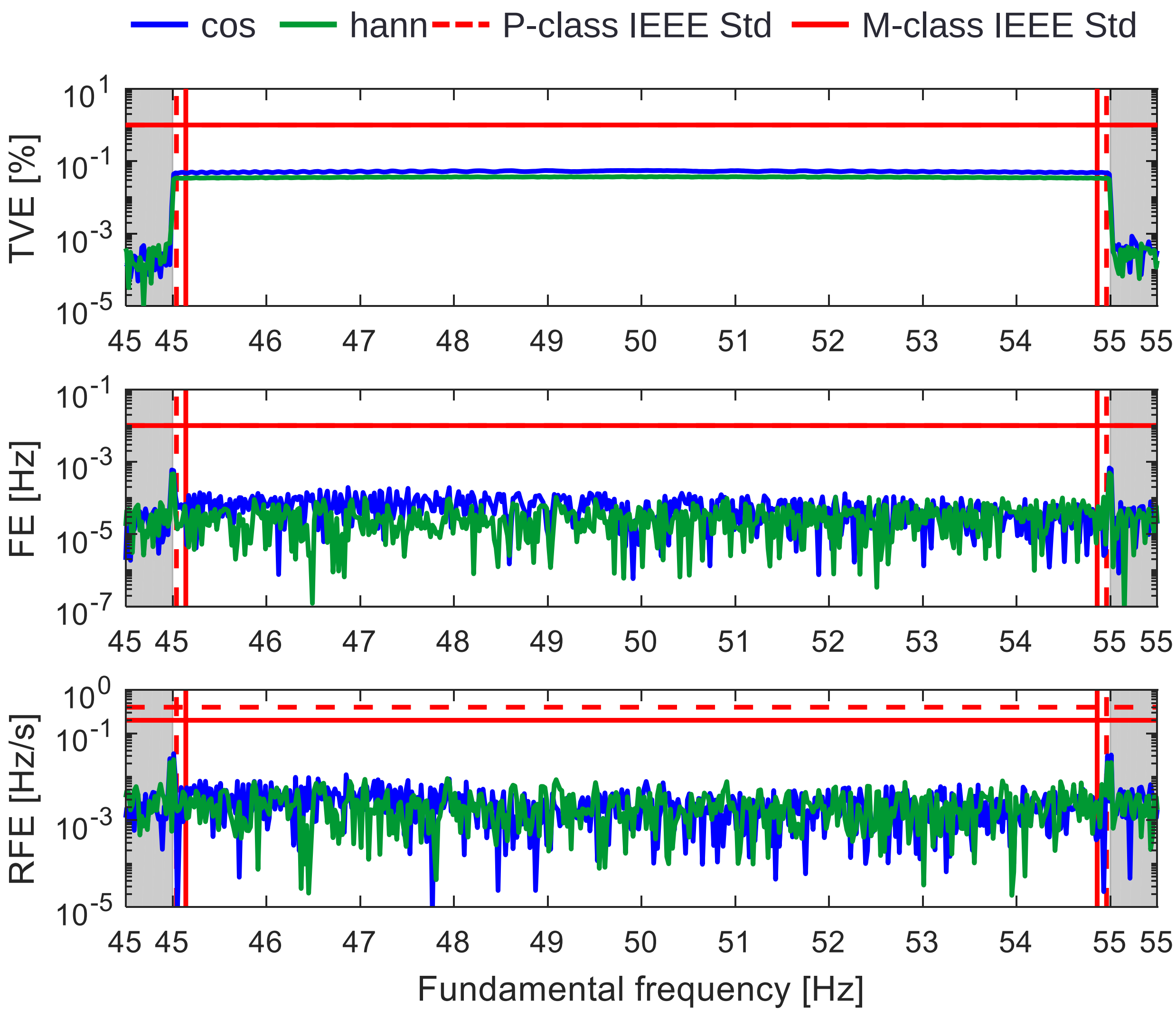
P = 2.3, M = 14 Hz/s

cos = 0.6 Hz/s

hann = 0.5 Hz/s

Performance Assessment

Dynamic conditions – Frequency Ramp (positive)



Max Errors:

TVE

P&M = 1%

cos = 0.06%

hann = 0.04%

FE

P&M = 10 mHz

cos = 0.2 mHz

hann = 0.2 mHz

RFE

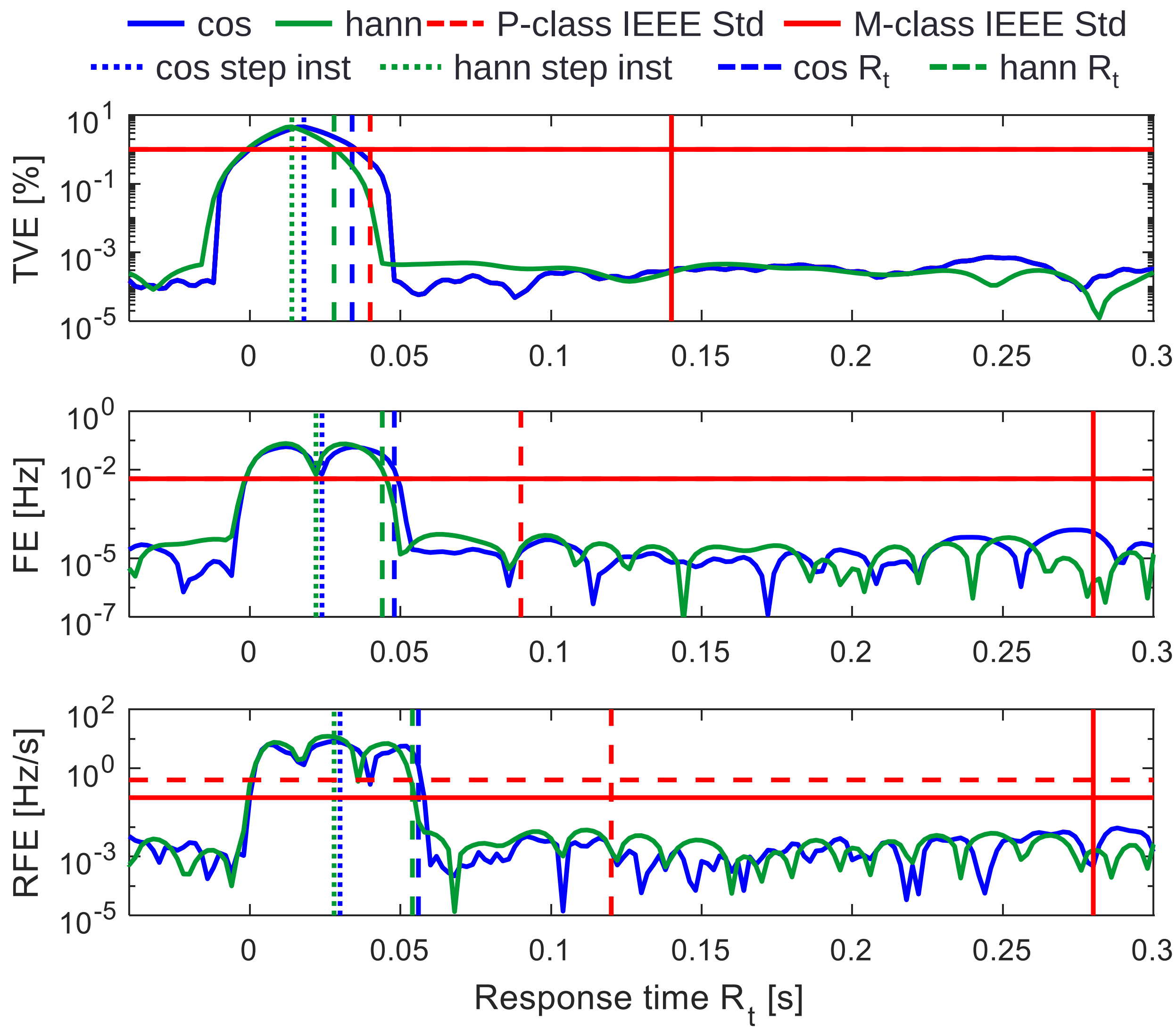
P = 0.4, M = 0.2 Hz/s

cos = 0.01 Hz/s

hann = 0.01 Hz/s

Performance Assessment

Dynamic conditions – Amplitude Step (positive)



Max Response time:

TVE

P = 0.04, M = 0.14 s

cos = 0.034 s

hann = 0.028 s

FE

P = 0.09, M = 0.28 s

cos = 0.048 s

hann = 0.044 s

RFE

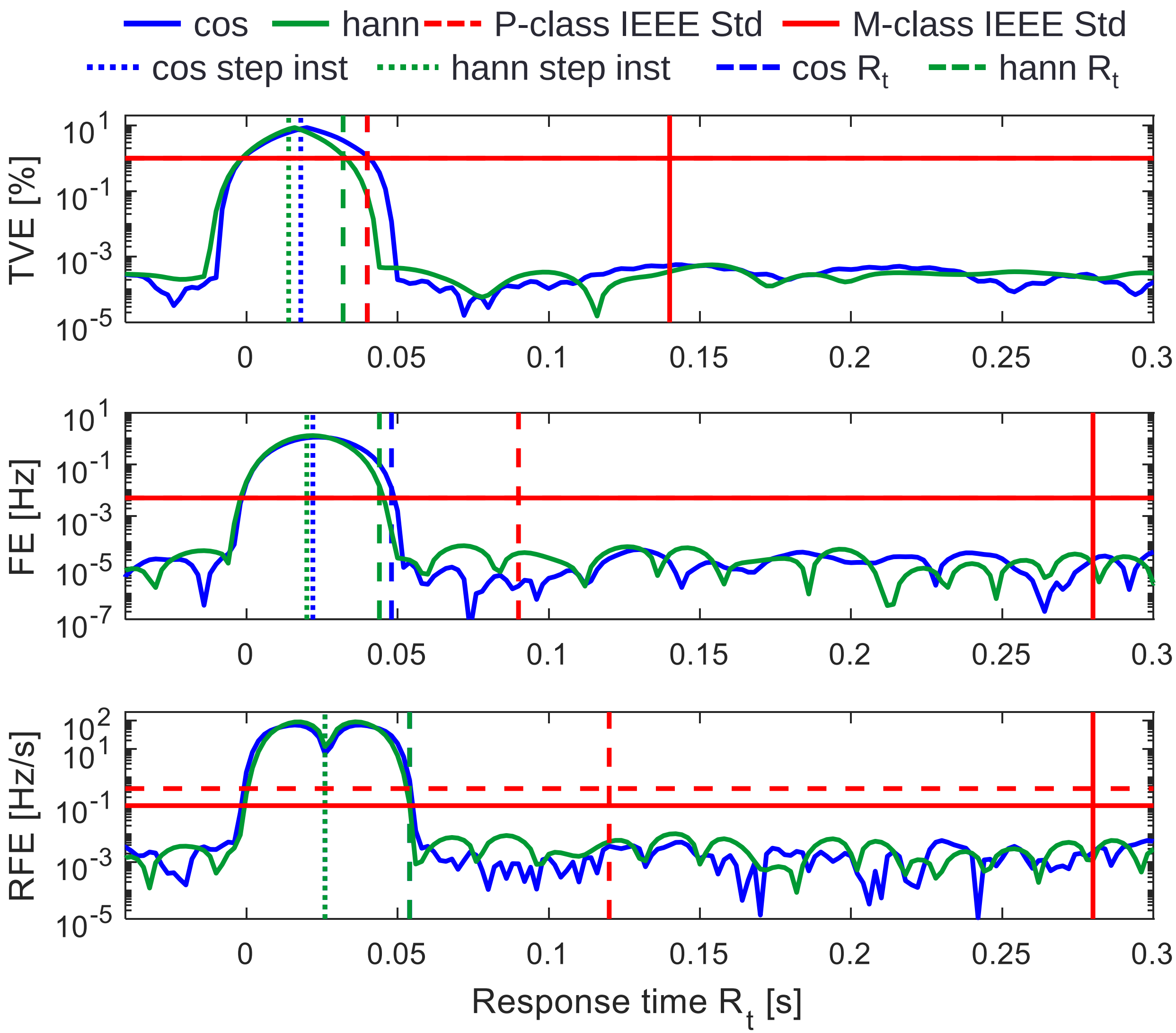
P = 0.12, M = 0.28 s

cos = 0.056 s

hann = 0.054 s

Performance Assessment

Dynamic conditions – Phase Step (positive)



Max Response time:

TVE

P = 0.04, M = 0.14 s

cos = 0.040 s

hann = 0.032 s

FE

P = 0.09, M = 0.28 s

cos = 0.048 s

hann = 0.044 s

RFE

P = 0.12, M = 0.28 s

cos = 0.054 s

hann = 0.054 s

Computational Complexity

Implementation on National Instruments Compact-Rio

- **e-IpDFT ➔** 20 μs [3]
- **i-IpDFT ➔** $2 \cdot Q \cdot 20$ μs ➔ 0.64 ms cos and 1.12 ms hann
- Compatible with the 10 ms time budget @ 50 fps
- Feasibility of the i-IpDFT technique implementation on an FPGA-based embedded device

1: $X(k) = \mathtt{DFT}(x(n))$
2: $\{\widehat{f}_0^0, \widehat{A}_0^0, \widehat{\varphi}_0^0\}|_P = \mathtt{e\!-\!IpDFT}[X(k)]$
3: $\widehat{X}_0^0(k) = \mathtt{wf}(\widehat{f}_0^0, \widehat{A}_0^0, \widehat{\varphi}_0^0) + \mathtt{wf}(-\widehat{f}_0^0, \widehat{A}_0^0, -\widehat{\varphi}_0^0)$
4: **if** $\sum |X(k) - \widehat{X}_0^0(k)|^2 > \lambda \cdot \sum |X(k)|^2$
5:     **for** $q = 1 \to Q$
6:         $\{\widehat{f}_i^q, \widehat{A}_i^q, \widehat{\varphi}_i^q\}|_P = \mathtt{e\!-\!IpDFT}[X(k) - \widehat{X}_0^{q-1}(k)]$
7:         $\widehat{X}_i^q(k) = \mathtt{wf}(\widehat{f}_i^q, \widehat{A}_i^q, \widehat{\varphi}_i^q) + \mathtt{wf}(-\widehat{f}_i^q, \widehat{A}_i^q, -\widehat{\varphi}_i^q)$
8:         $\{\widehat{f}_0^q, \widehat{A}_0^q, \widehat{\varphi}_0^q\}|_P = \mathtt{e\!-\!IpDFT}[X(k) - \widehat{X}_i^q(k)]$
9:         $\widehat{X}_0^q(k) = \mathtt{wf}(\widehat{f}_0^q, \widehat{A}_0^q, \widehat{\varphi}_0^q) + \mathtt{wf}(-\widehat{f}_0^q, \widehat{A}_0^q, -\widehat{\varphi}_0^q)$
10:     **end for**
11: **else**
12:     **break**
13: **end if**

	i-IpDFT Variable	Value
	K	11
	P	2
	Q	16 (cos), 28 (Hann)
Function	$+ \mid - \mid \times$	$\div \mid exp \mid sin$
IpDFT	14	3
wf (cos)	$18K$	$11K$
wf (Hann)	$23K$	$16K$
e-IpDFT	$(P{+}1) \cdot \mathtt{IpDFT} + P \cdot \mathtt{wf} + KP$	$(P{+}1) \cdot \mathtt{IpDFT} + P \cdot \mathtt{wf}$
Alg. II	$+ \mid - \mid \times$	$\div \mid exp \mid sin$
line 2	e-IpDFT	e-IpDFT
line 3	$K + 2 \cdot \mathtt{wf}$	$2 \cdot \mathtt{wf}$
line 4	$5K - 2$	-
line 6, 8	$Q \cdot \mathtt{e\!-\!IpDFT} + K$	$Q \cdot \mathtt{e\!-\!IpDFT}$
line 7, 9	$Q \cdot (K + 2 \cdot \mathtt{wf})$	$Q \cdot (2 \cdot \mathtt{wf})$

Conclusions & Future Works

- IpDFT-based SE algorithm for P and M-class compliant PMUs
- **First IpDFT-based synchrophasor estimation technique resilient to OOBI**
- During OOBI test the **maximum TVE** is **0,1%** (being 1,3% the maximum allowed limit) and the **maximum FE** is **4 mHz** (being 10 mHz the maximum allowed limit)

- Patent submitted to PCT
- Implementation of the i-IpDFT in an embedded device (National Instruments Compact-Rio embedded control and acquisition system)

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References

1. A. Derviškadić, P. Romano and M. Paolone, "Iterative-interpolated DFT for Synchrophasor Estimation: a Single Algorithm for P and M-class compliant PMUs," *IEEE Transactions on Instrumentation and Measurement.*
2. P. Romano and M. Paolone, "Enhanced interpolated-DFT for synchrophasor estimation in FPGAs: Theory, implementation, and validation of a PMU prototype," *IEEE Transactions on Instrumentation and Measurement*, vol. 63, no. 12, pp. 2824–2836, Dec 2014.
3. P. Romano and M. Paolone, "An enhanced interpolated-modulated sliding DFT for high reporting rate PMUs," in *2014 IEEE International Workshop on Applied Measurements for Power Systems Proceedings (AMPS)*, Sept 2014, pp. 1–6.