

**Simulation of Wide Area Frequency
Measurements from
Phasor Measurement Units (PMUs)
or
Frequency Disturbance Recorders (FDRs)**

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Outline

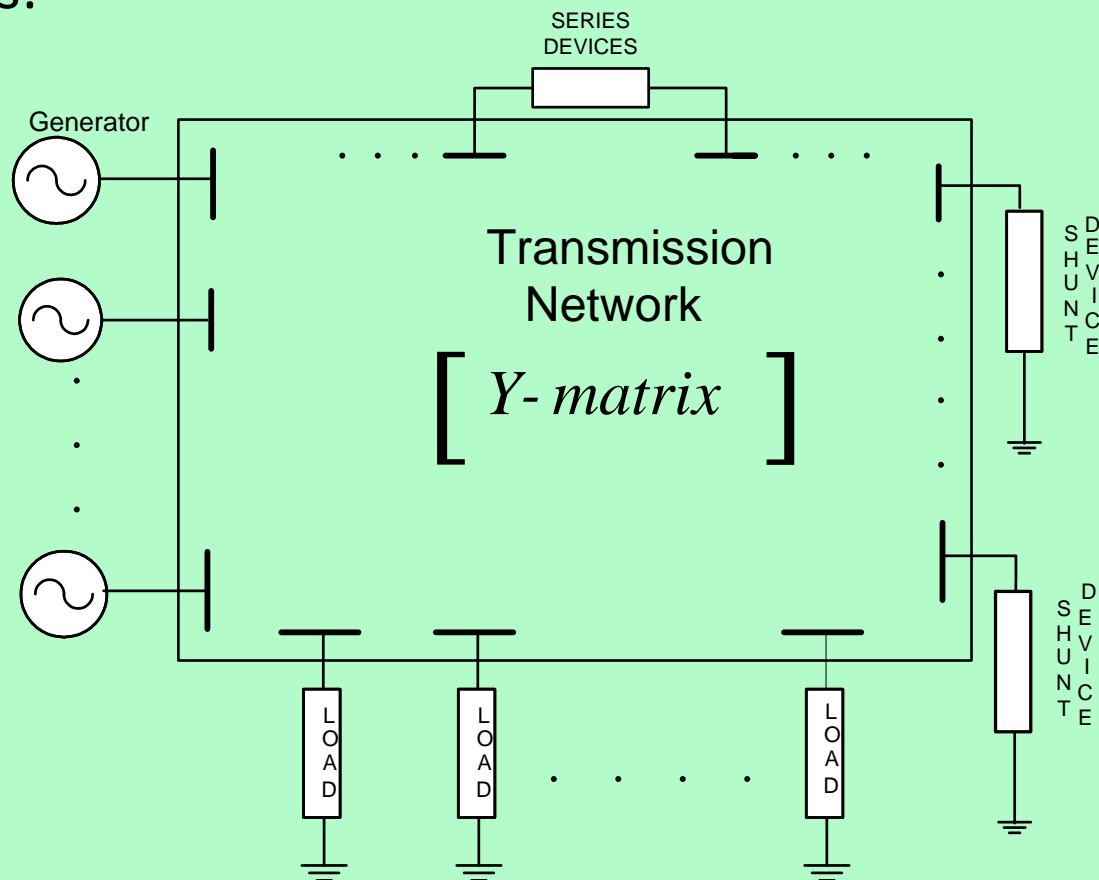
- Motivation for the Study
- What is Meant by Power System Frequency at Certain Bus/Location?
- Frequency Measured by PMUs/FDRs
- Simulation of Frequency Measured by PMUs/FDRs
- Proposed Methods for Simulation of Frequency Measured by PMUs/FDRs
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Motivation for the Study

- It is anticipated that PMUs and FDRs will be integrated with power system for real time control:
 - Power System Stabilizer (PSS)
 - Automatic Load Frequency Control (ALFC)
- It would be desired to simulate the system performance before the actual implementation:
 - In actual implementation we use the measurements from FDRs and/or PMUs
 - What should we use for those measurements in simulation phase?
- We propose a method of estimating the system frequency at certain bus / location that simulates the measured frequency by PMUs / FDRs

What is Meant by Power System Frequency at Certain Bus/Location?

- Symbolic Representation of a multi-machine Power Systems:



What is Meant by Power System Frequency at Certain Bus/Location? (Cont.)

- The voltage at a generic bus is contributed by all generators in the power system. Thus this voltage involves the frequencies of all generators
- Question:
 - What is meant by power system frequency at certain location/bus?
 - Which frequency is measured by PMUs / FDRs?

What is Meant by Power System Frequency at Certain Bus/Location? (Cont.)

- Answer:
 - **In Steady State** all generators have the same frequency (f_{ss} Hz). The i^{th} machine may be considered as a voltage source of:

$$e_i(t) = E_i \cos(2\pi f_{ss} t + \delta_i)$$

- Thus, the voltage at all points of the power system will have the same frequency (f_{ss}) and this is the frequency measured by PMUs and/or FDRs.

What is Meant by Power System Frequency at Certain Bus/Location? (Cont.)

- **Upon a disturbance** the power system enters into a dynamic state and different generators will run with different frequencies that vary with respect to time.

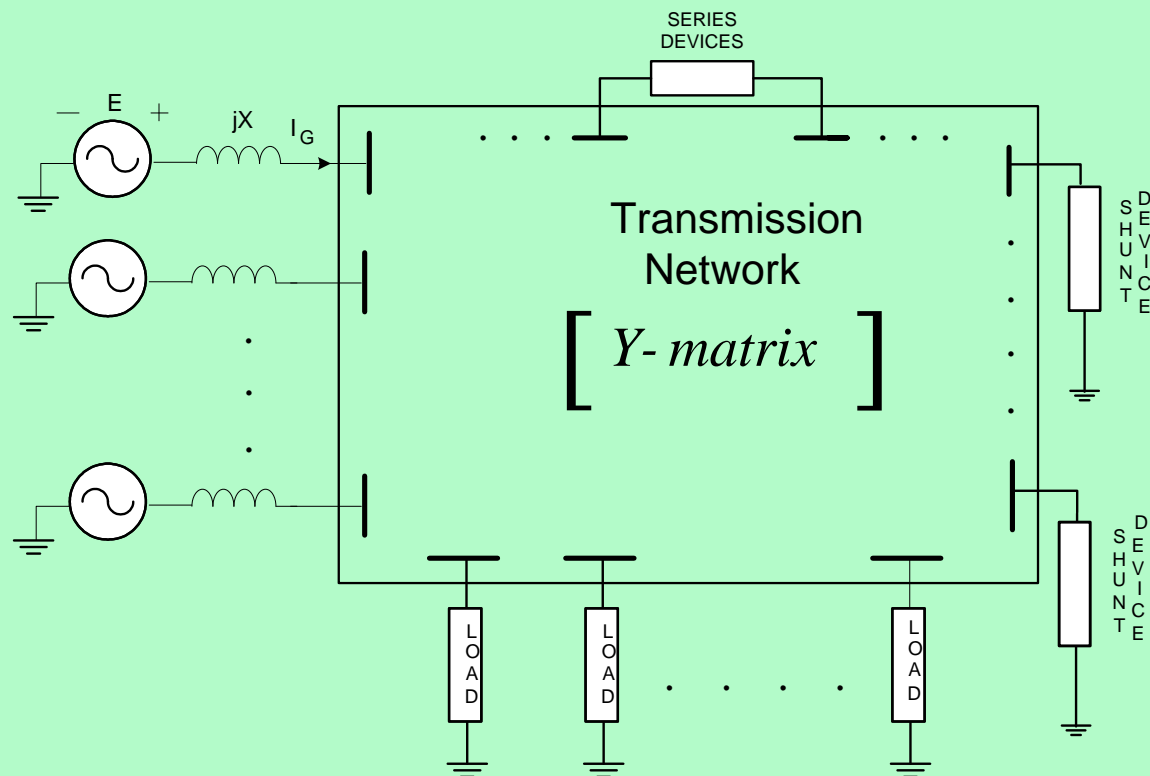
In this case, a generator may be considered as a voltage source of:

$$e_i(t) = E_i \cos(2\pi f_i t + \delta_i)$$

where, E_i , f_i , and δ_i are slow varying functions of time. These time functions may be found by solving the Differential Algebraic Equations (DAE) representing the power system.

What is Meant by Power System Frequency at Certain Bus/Location? (Cont.)

At a given time t , for a small interval or window of time ($\Delta t = n$ cycles), E_i , f_i , and δ_i may be assumed constants. During this interval, the power system can be represented as:



What is Meant by Power System Frequency at Certain Bus/Location? (Cont.)

Now we have a circuit with several voltage sources of different frequencies. The actual voltage at Bus i , using superposition theorem becomes:

$$\begin{aligned}v_i^{act}(t) &= v_{i,1}(t) + \cdots + v_{i,N_G}(t) = \sum_{j=1}^{N_G} v_{i,j}(t) \\ &= V_{i,1} \cos(2\pi f_1 t + \theta_{i,1}) + \cdots + V_{i,N_G} \cos(2\pi f_{N_G} t + \theta_{i,N_G}) \\ &= \sum_{j=1}^{N_G} V_{i,j} \cos(2\pi f_j t + \theta_{i,j})\end{aligned}$$

where, $v_{i,j}(t)$ represents the voltage at Bus i due to generator j .

What is Meant by Power System Frequency at Certain Bus/Location? (Cont.)

The equation shows that the voltage at Bus i is a multi-frequency voltage. These frequencies are all close to 60 Hz.

Now, even though the voltage at Bus i involves many frequencies, we approximate this voltage to an estimated single-frequency voltage (this approximation is acceptable for dynamic studies of power systems) defined as follows:

$$v_i^{est}(t) = V_i^{est} \cos(2\pi f_i^{est} t + \theta_i^{est})$$

The frequency f_i^{est} represents the frequency of the system at this location / bus and equals the frequency measured by PMUs / FDRs.

Frequency Measured by PMUs/FDRs

- PMUs and FDRs implicitly assume that the estimated voltage signal is equal to the actual signal:

$$v_i^{est}(t) = v_i^{act}(t) = \sum_{j=1}^{N_G} v_{i,j}(t)$$

or

$$V_i^{est} \cos(2\pi f_i^{est} t + \theta_i^{est}) = \sum_{j=1}^{N_G} V_{i,j} \cos(2\pi f_j t + \theta_{i,j})$$

It should be noted that, the above equality is not possible and it is only an approximation

Frequency Measured by PMUs/FDRs (Cont.)

$$v_i^{est}(t) = v_i^{act}(t) = \sum_{j=1}^{N_G} v_{i,j}(t)$$

The FDRs/PMUs do not know the individual terms in the right hand side of the equation; they only have access to the sampled value of the total (or actual) signal, $v_i^{act}(t)$, which they use to calculate f_i^{est} .

The measurements for a certain interval (or window) of time, ($\Delta t = n$ cycles) are used to find the value of f_i^{est} for the entire duration of Δt .

Simulation of Frequency Measured by PMUs/FDRs

Simulation of the PMUs/FDRs measurements is also based on the same equation as used by PMUs/FDRs:

$$v_i^{est}(t) = v_i^{act}(t) = \sum_{j=1}^{N_G} v_{i,j}(t)$$

or

$$V_i^{est} \cos(2\pi f_i^{est} t + \theta_i^{est}) = \sum_{j=1}^{N_G} V_{i,j} \cos(2\pi f_j t + \theta_{i,j})$$

Unlike the real world measurements, in simulation domain we know every term in the right hand side of the equation using which we try to find the best value of the estimated frequency, f_i^{est} .

Simulation of Frequency Measured by PMUs/FDRs (Cont.)

The best estimate is one that minimizes the Integral of Squared Error (ISE) defined as follows:

$$ISE = \int_{t_o}^{t_o + \Delta t} \left(v_i^{est}(t) - v_i^{act}(t) \right)^2 dt$$

Three methods are examined and compared with each other based on their associated errors.

Proposed Methods for Simulation of Frequency Measured by PMUs/FDRs

Method 1- Brute Force:

- Assuming the actual voltage is identical to the estimated voltage as follows:

$$v_i^{act}(t) = V_i^{est} \cos(2\pi f_i^{est} t + \theta_i^{est})$$

then we have:

$$f_i^{est} = \frac{1}{2\pi t} \left[\cos^{-1} \left(\frac{v_i^{act}(t)}{V_i^{est}} \right) - \theta_i^{est} \right]$$

- The above equation can be used for several instants of time during the interval of interest. The average will be taken and chosen to be the estimated frequency

Proposed Methods for Simulation of Frequency Measured by PMUs/FDRs (Cont.)

Method 2--Four-Point Total Square Error Minimization:

- In this method, f_i^{est} is found by minimizing the sum of the square error at four points. These points are $t = T/4$, $t = T/2$, $t = 3T/2$, $t = 2T$, where $T = 1/60$. Where the Square Error (SE) at a given time, t_k , is found by:

$$SE(t_k) = \left(v_i^{act}(t_k) - v_i^{est}(t_k) \right)^2$$

- Thus the Total Square Error (TSE) is given by the following:

$$TSE(t) = SE(t_1) + SE(t_2) + SE(t_3) + SE(t_4)$$

- The minimum total square error is found by taking the first derivate of $TSE(t)$, setting it equal to zero, and solving for f_i^{est} .

Proposed Methods for Simulation of Frequency Measured by PMUs/FDRs (Cont.)

Method 3- Proportional Voltage-Frequency:

- In this method, f_i^{est} is estimated using the following equation:

$$V_i^{est} \cos(2\pi f_i^{est} t + \theta_i^{est}) = \sum_{j=1}^{N_G} V_{i,j} \cos(2\pi f_j t + \theta_{i,j})$$

- It is assumed that the contribution of each of the components on the right hand side to f_i^{est} is proportional to its magnitude:

$$f_i^{est} = \sum_{j=1}^{N_G} \frac{V_{i,j} \cos(\theta_i^{est} - \theta_{i,j})}{V_i^{est}} f_j$$

Proposed Methods for Simulation of Frequency Measured by PMUs/FDRs (Cont.)

Comparison of the Methods:

The table shows a comparison of different proposed methods in terms of Integral of Squared Error (ISE) and Normalized Error (NE) :

Frequency Estimation Method	Integral of Squared Error (ISE)	Normalized Error
Brute Force	5.946E-16	1.8888E-07
Four-Point Total Square Error Minimization	6.2675E-16	1.9392E-07
Proportional Voltage-Frequency	6.2985E-16	1.944E-07

Some Comments

Frequency Wave Propagation:

- The frequency measurement from PMUs/FDRs has the characteristics of a traveling wave which shows the impacts of a disturbance travels with a certain speed. This speed (although less than the speed of light) is large enough for the impacts to be considered instantaneous for dynamic behavior studies of power systems.
- In simulation domain, the impact of any disturbance in the system is felt by any part of the system instantly. This is due to the fact that the transmission system is modeled as a static system by its Y-matrix. The magnitude of the impact may be different for different locations at different times, similar to PMUs/FDRs measurements.

Some Comments (Cont.)

Bus Frequency Calculated by PSS/E:

- PSS/E also calculates a frequency for each bus.
- When plotted for the simulations we performed, it seems that the frequencies of bus-voltages estimated by PSS/E are unrealistically different than the frequencies of the generators which are responsible for those voltages.

Some Comments (Cont)

Equivalent Inertial Center Frequency:

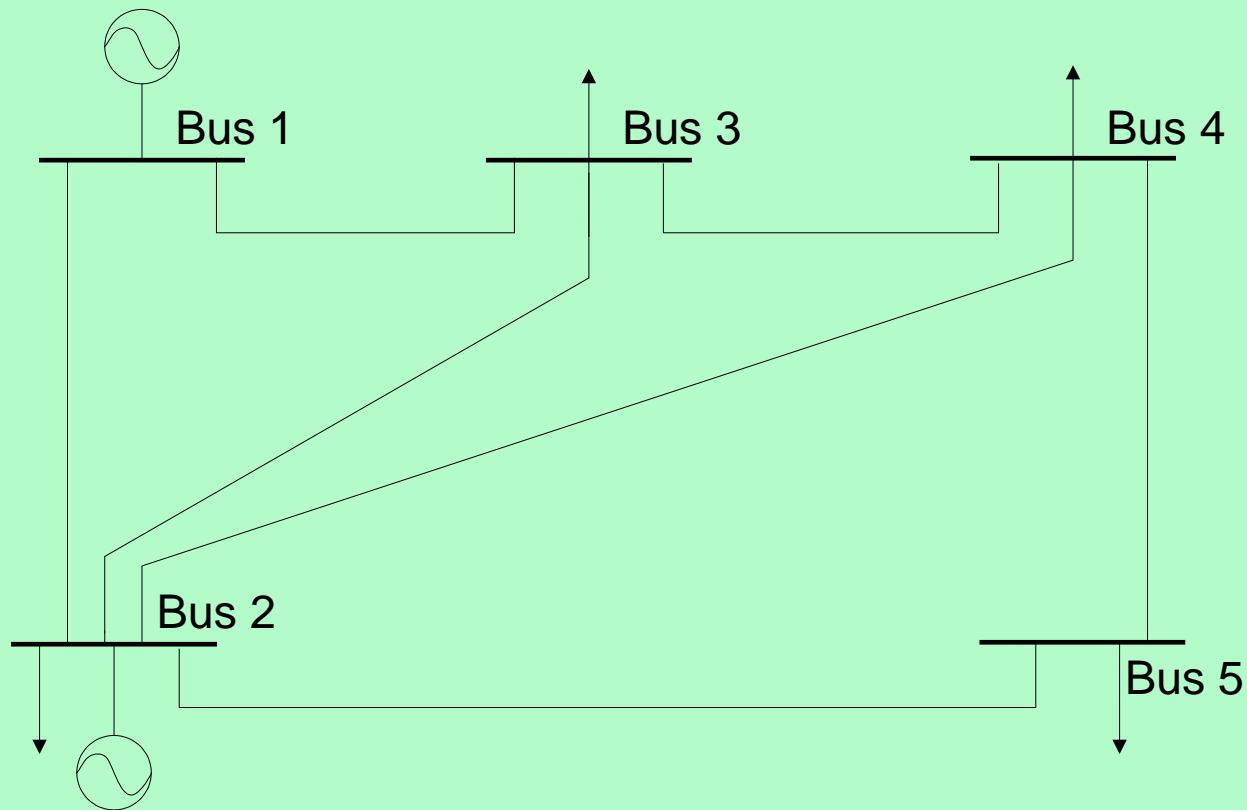
The equivalent inertial center frequency of a system f_c is a single value representing (approximately) the frequency of the entire system.

It is calculated by taking a weighted average of machine frequencies and machine inertias as follows:

$$f_c = \sum_{i=1}^{N_G} \frac{H_i f_i}{H_i}$$

The center frequency is only a function of time (not location / bus) and is, thus, the same for all buses in the system considered.

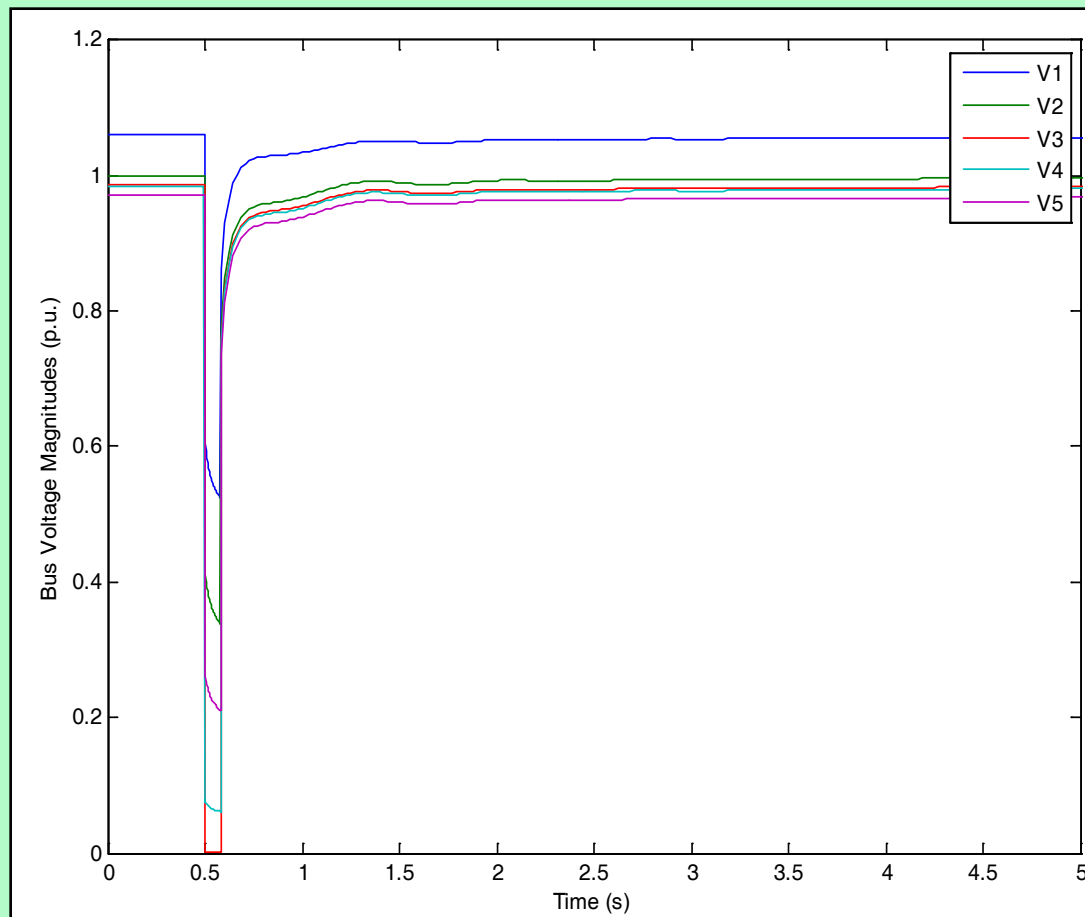
Five-Bus Example System



A 5-cycle, three-phase short-circuit fault is applied to Bus 3

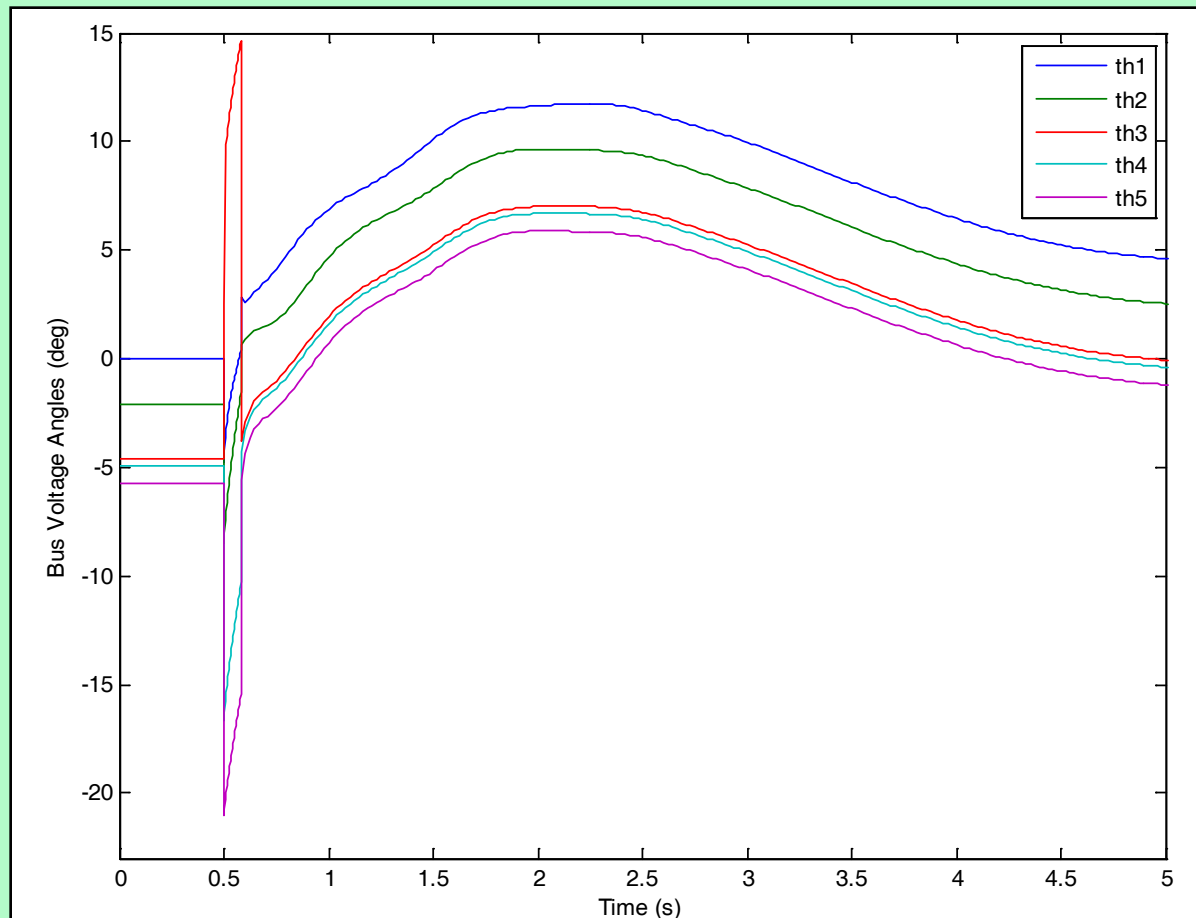
Five-Bus Example System (Cont.)

- Bus Voltage Magnitude from Simulation



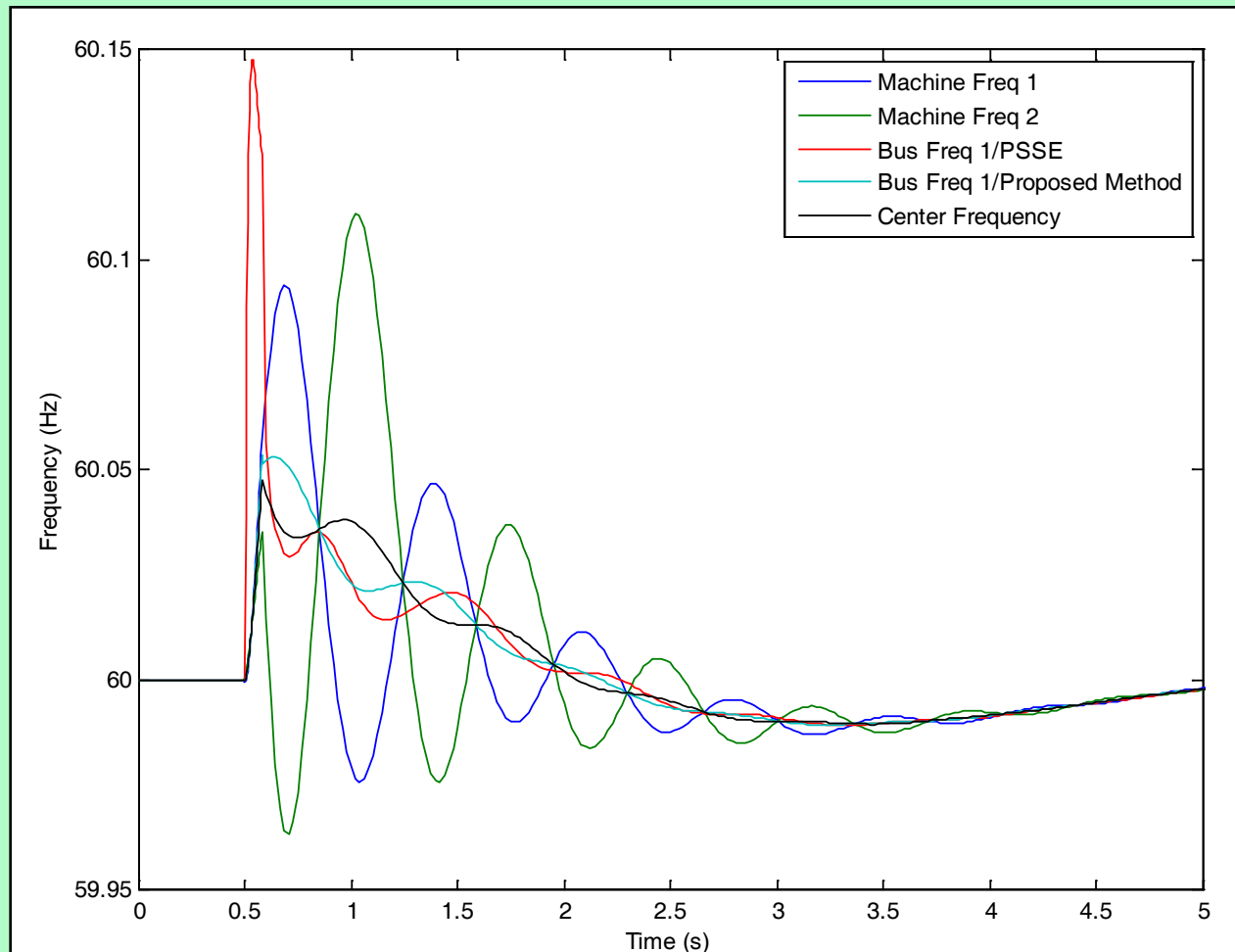
Five-Bus Example System (Cont.)

- Bus Voltage Angle from Simulation



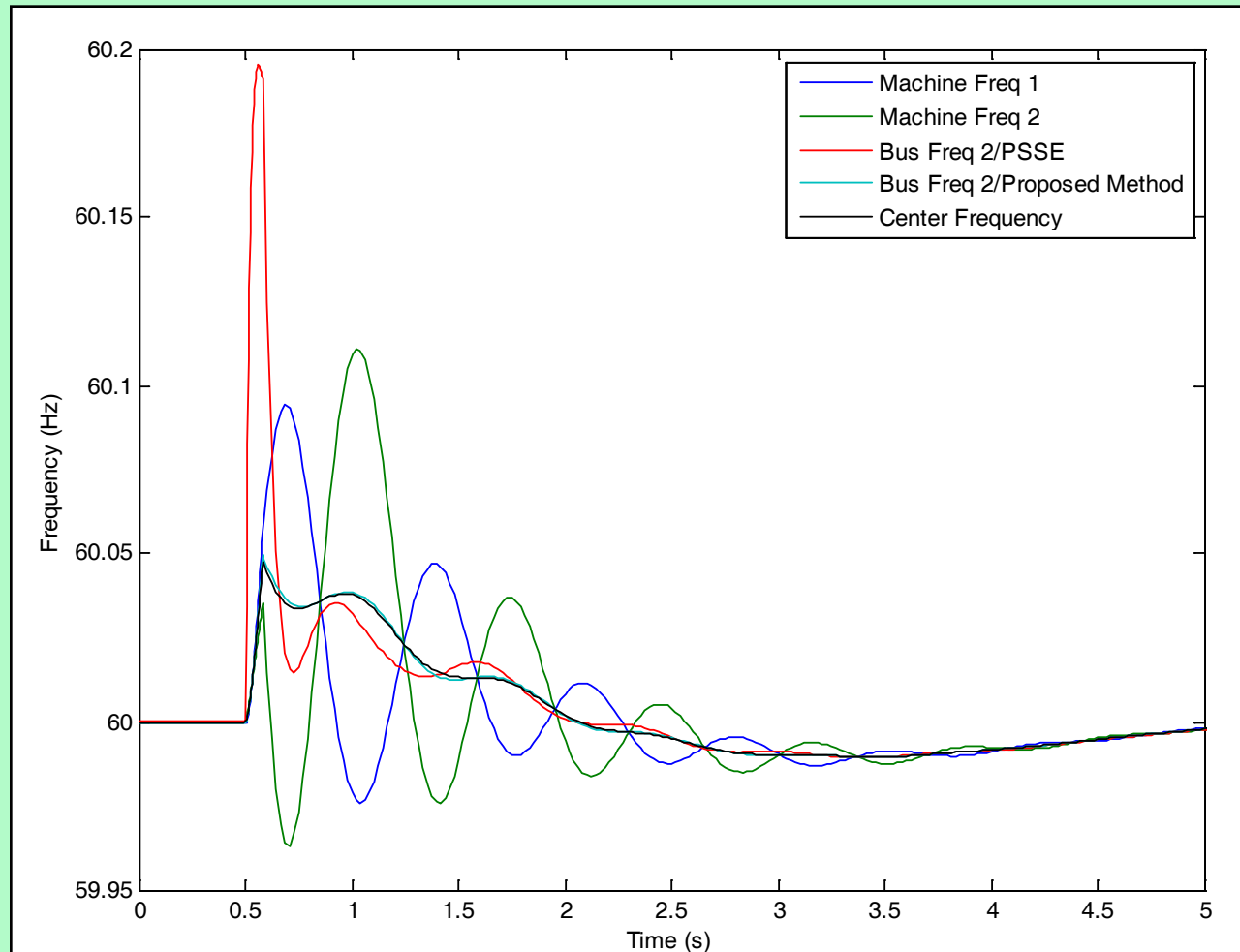
Five-Bus Example System (Cont.)

- Frequency Simulation for Bus 1



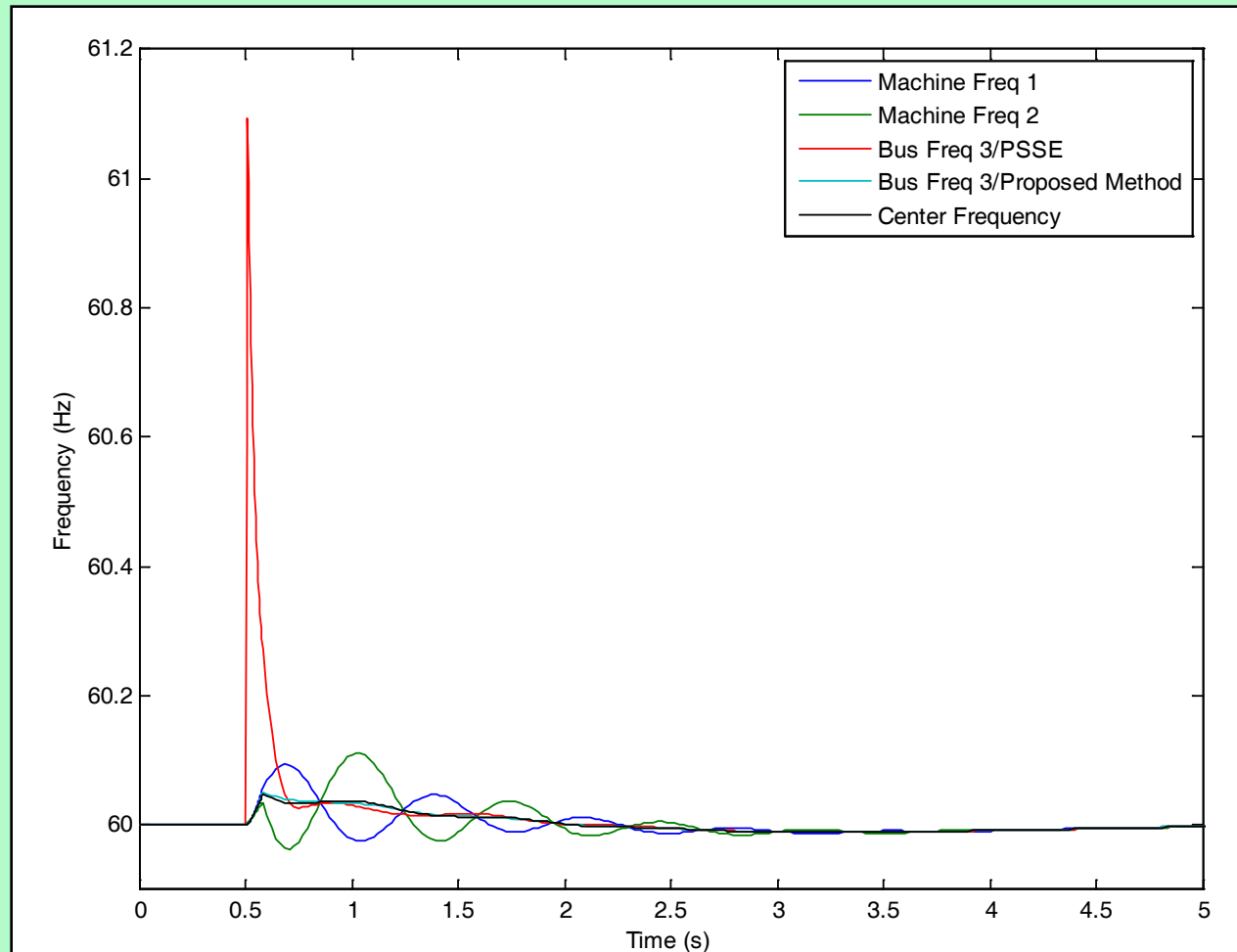
Five-Bus Example System (Cont.)

- Frequency Simulation for Bus 2



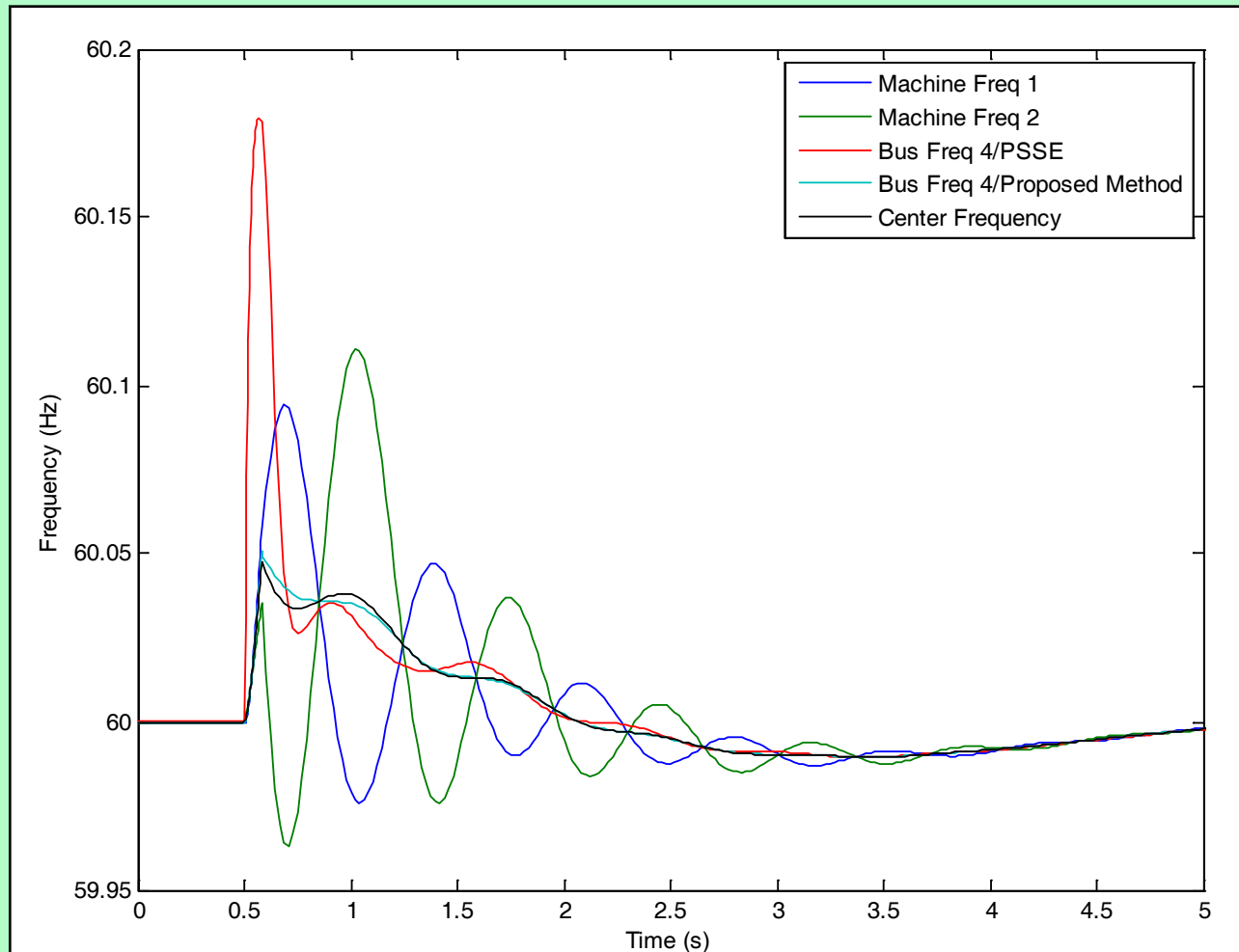
Five-Bus Example System (Cont.)

- Frequency Simulation for Bus 3



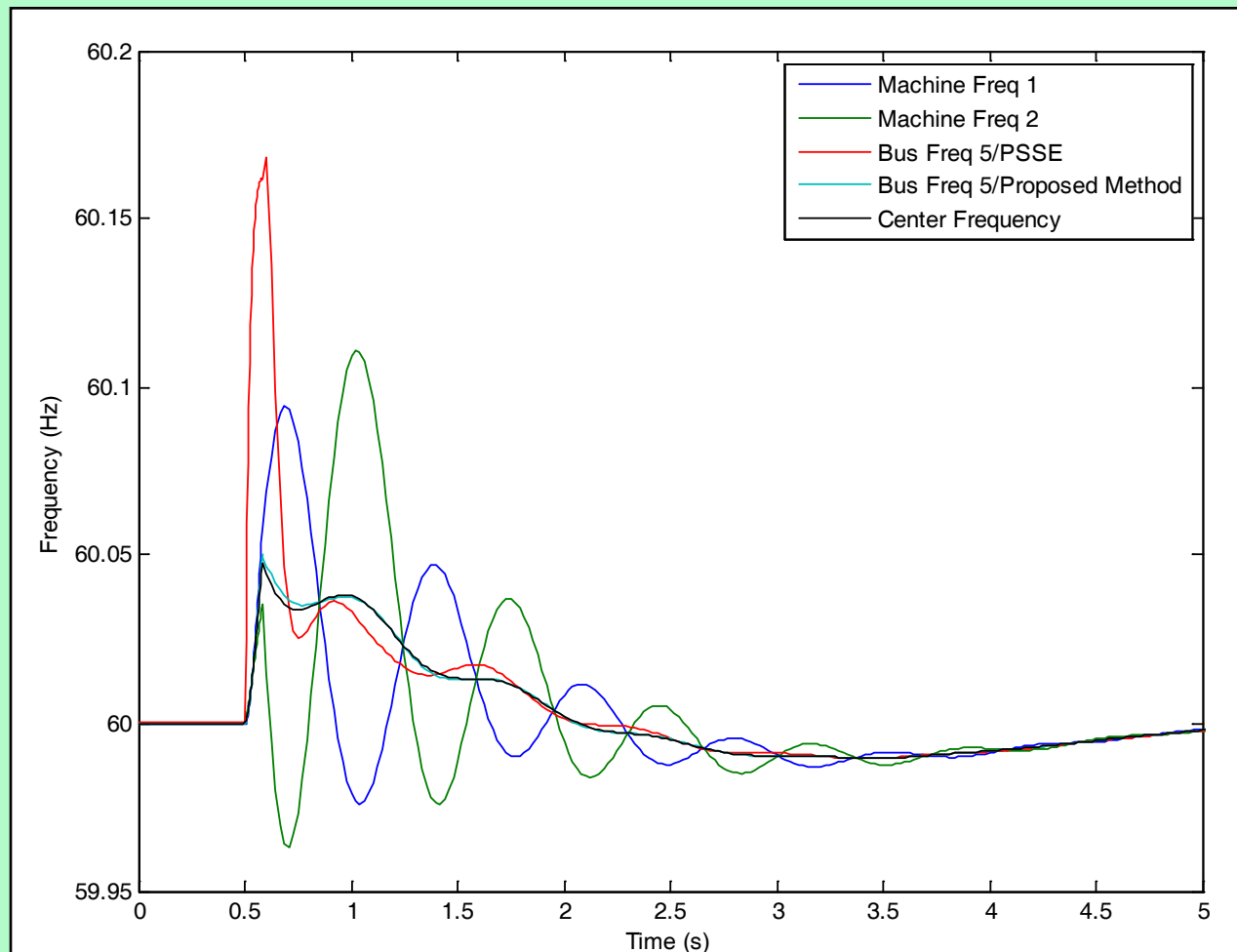
Five-Bus Example System (Cont.)

- Frequency Simulation for Bus 4

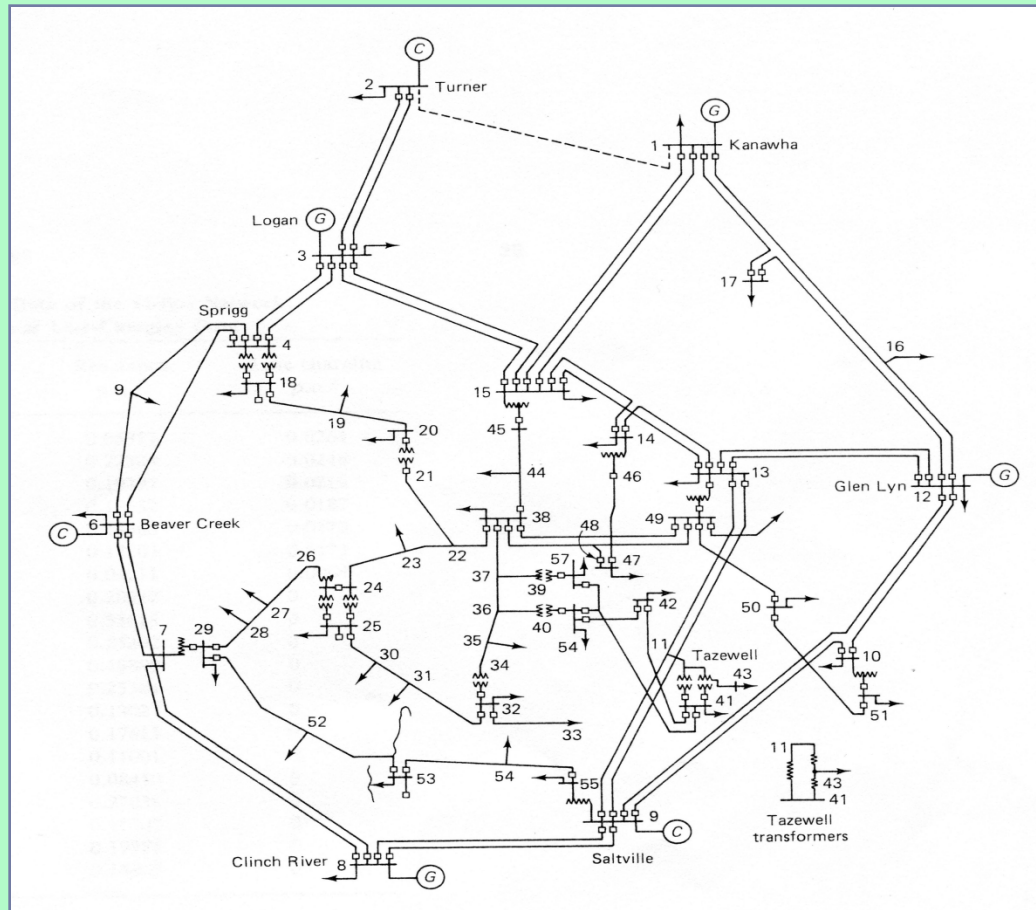


Five-Bus Example System (Cont.)

- Frequency Simulation for Bus 5



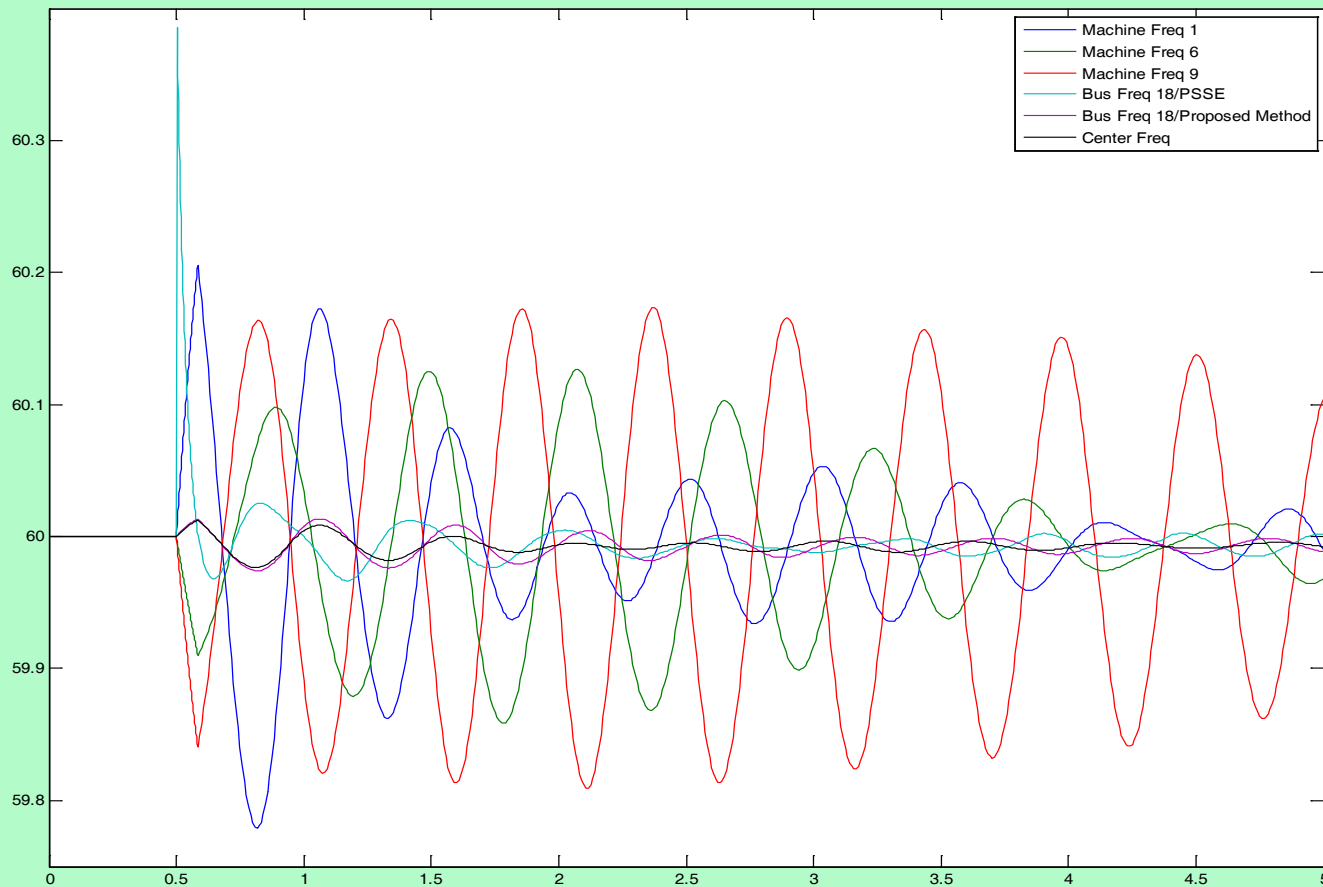
Other Test Systems



IEEE 57-bus System: A 5-cycle, three-phase short-circuit fault is applied to Bus 18

Other Test Systems

Frequency Simulation for Bus 18



Discussion

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