Power System Research Consortium
PSRC

PHASOR STATE ESTIMATION

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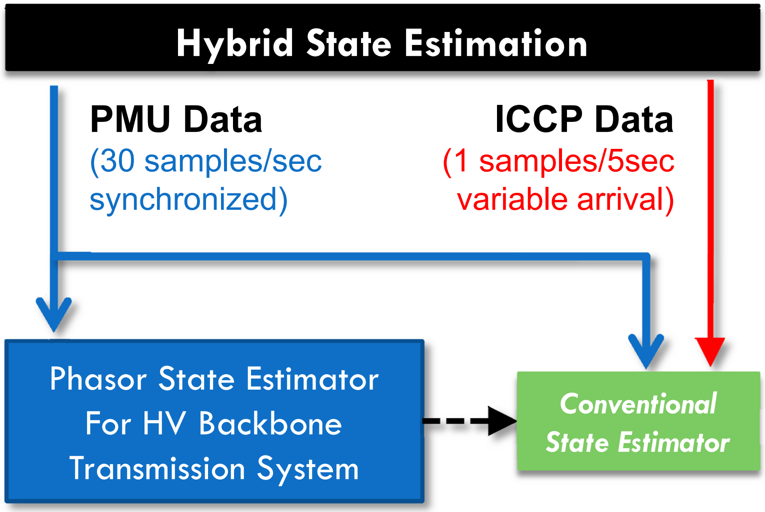
Outline

- Phasor State Estimation (PSE)
  - PSE Concept
  - PSE with and without Redundancy
- PSE Model and Implementation
  - Problem Formulation
  - Weighted Nonlinear Least Squares
  - Application to the Marcy-Massena Transfer Path
- Angle-Bias Correction with PSE
  - Angle-Bias Errors on PDC PMU Data
  - Angle-Bias Detection Extension of the PSE Model
- Further Work
- References

Phasor State Estimation (i)

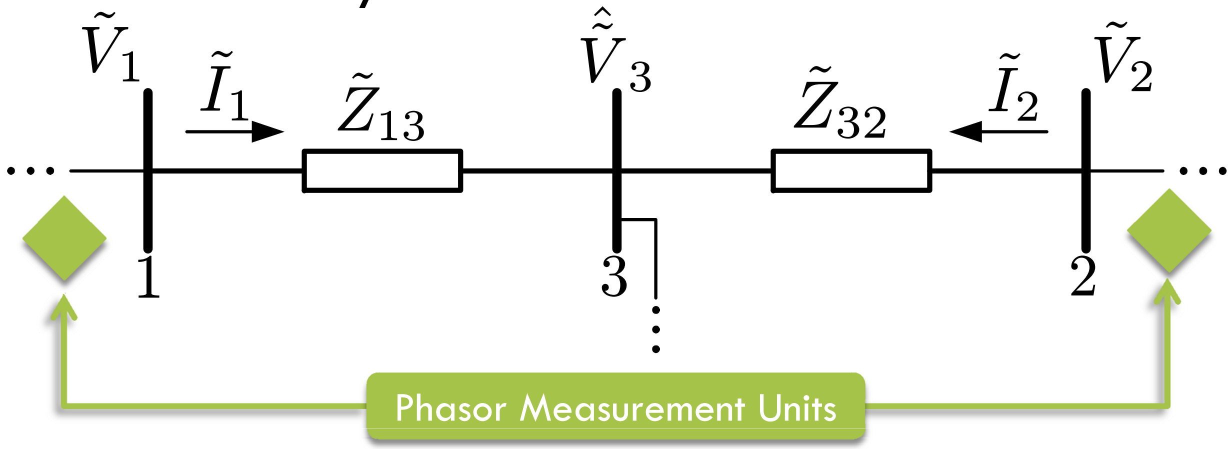
- Concept and Applications
  - Hybrid State Estimation



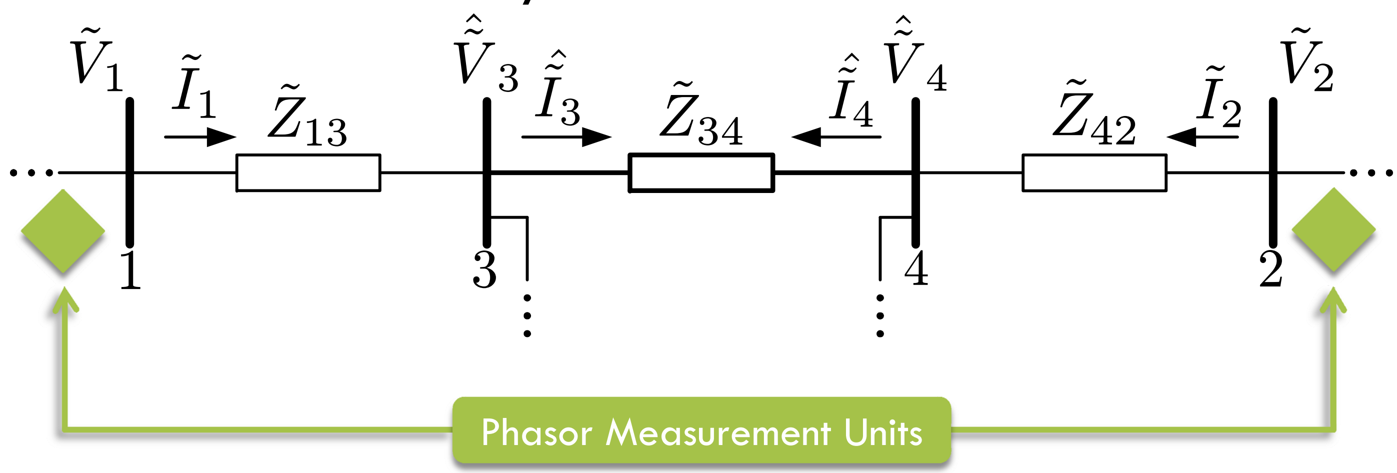
  - Phasor State Estimator is independent of the SE, and supported only by a limited number of PMUs
    - Added reliability when the SE is functional
    - Provide observability of the HV network when the SE is not available.
    - Functions:
      - Provide estimates for unmonitored buses
      - Detect bad PMU data (Angle-biases)
      - Provide a platform to develop monitoring and control applications

Phasor State Estimation (ii)

- PSE with Redundancy



- PSE without Redundancy

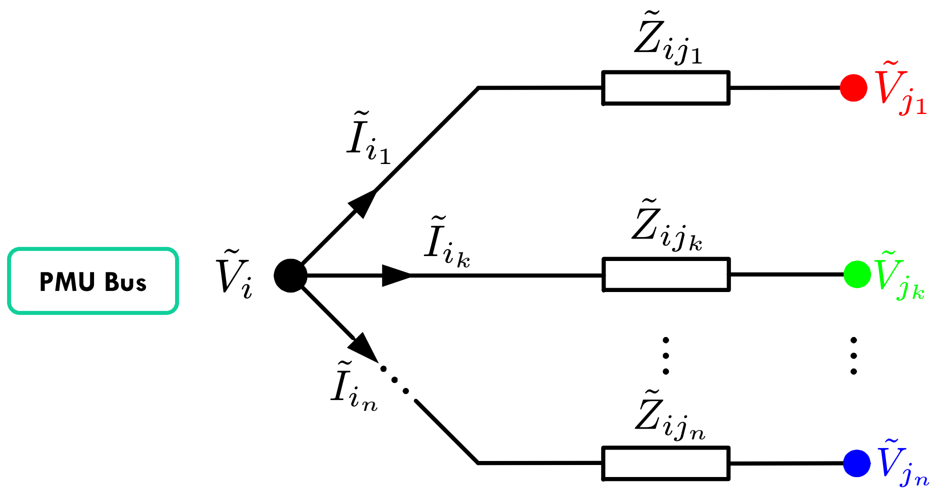


Problem Formulation (i)

- We formulate the problem as a Weighted Least Squares problem.
- This requires to construct a model containing:
  - The Synchrophasor Measurements, and
  - The network model (transmission lines, transformers, etc.)
- The state-variables of the model and the measurements should be explicit in the formulation.
- Sum of squares solution should be reliable and fast.
- Inherent data errors should be be detected and corrected.

Problem Formulation (ii)

- **Measurement Model:**
  - We construct the voltage and current magnitude, and angle equations at each PMU-measured Bus (i) as follows:



$V_i = V_{im}$ Voltage Magnitude Measurement

$I_i = I_{im}$ Current Magnitude Measurement

$\theta_i = \theta_{im}$ Voltage Angle Measurement

$\delta_i = \delta_{im}$ Current Angle Measurement

Problem Formulation (iii)

- **Network Model:**
  - Complex KVL Network Equations for each PMU-Bus:

$$\text{For } k = 1 \text{ to } N$$
$$\tilde{V}_{i_k} = \tilde{V}_{j_k} + \tilde{Z}_{ij_k}\tilde{I}_{i_k}$$

  - Separate the equation in real and imaginary parts.
  - This generates two network equations per PMU-Bus.
- Use Least Squares to estimate $\mathbf{x}$ from the network and measurement equations.

$$\mathbf{x} = \left[\begin{array}{cc|cc|cc|cc} V_i & I_i & V_{j_k} & I_{j_k} & \theta_i & \delta_{i_k} & \theta_{j_k} & \delta_{j_k} \end{array}\right]^T$$

  - A linear weighted LS can be designed, but it will not solve directly for the states above → does not relate directly to the measurements.
  - A weighted nonlinear least squares is formulated.

Weighted Nonlinear Least Squares (i)

- **Nonlinear Least Squares:**
  - Sum of squares measure for data fitting

$$\min_{\mathbf{x}(t)} q(\mathbf{x}(t)) = \frac{1}{2}\sum_{i=1}^{m}\left[w_i f_i\left(\mathbf{x}(t)\right)\right]^2, \quad \mathbf{x} \in R^n$$

  - Where $f_i(\mathbf{x}(t))$ are the network and measurement equations
- Iterative Gauss-Newton Solution:
  - Descent Direction:

$$\mathbf{H} = \left[\left\{\mathbf{W}\,\mathbf{J}\left(\mathbf{x}^{(k)}\right)\right\}^T \mathbf{W}\,\mathbf{J}\left(\mathbf{x}^{(k)}\right)\right]^{-1}$$

$$\mathbf{s}^{(k)} = -\mathbf{H}\left(\mathbf{W}\,\mathbf{J}\left(\mathbf{x}^{(k)}\right)\right)^T \mathbf{W}\,\mathbf{f}\left(\mathbf{x}^{(k)}\right)$$

  - Solution Update:

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \mathbf{s}^{(k)}$$

k denotes the number of iteration.

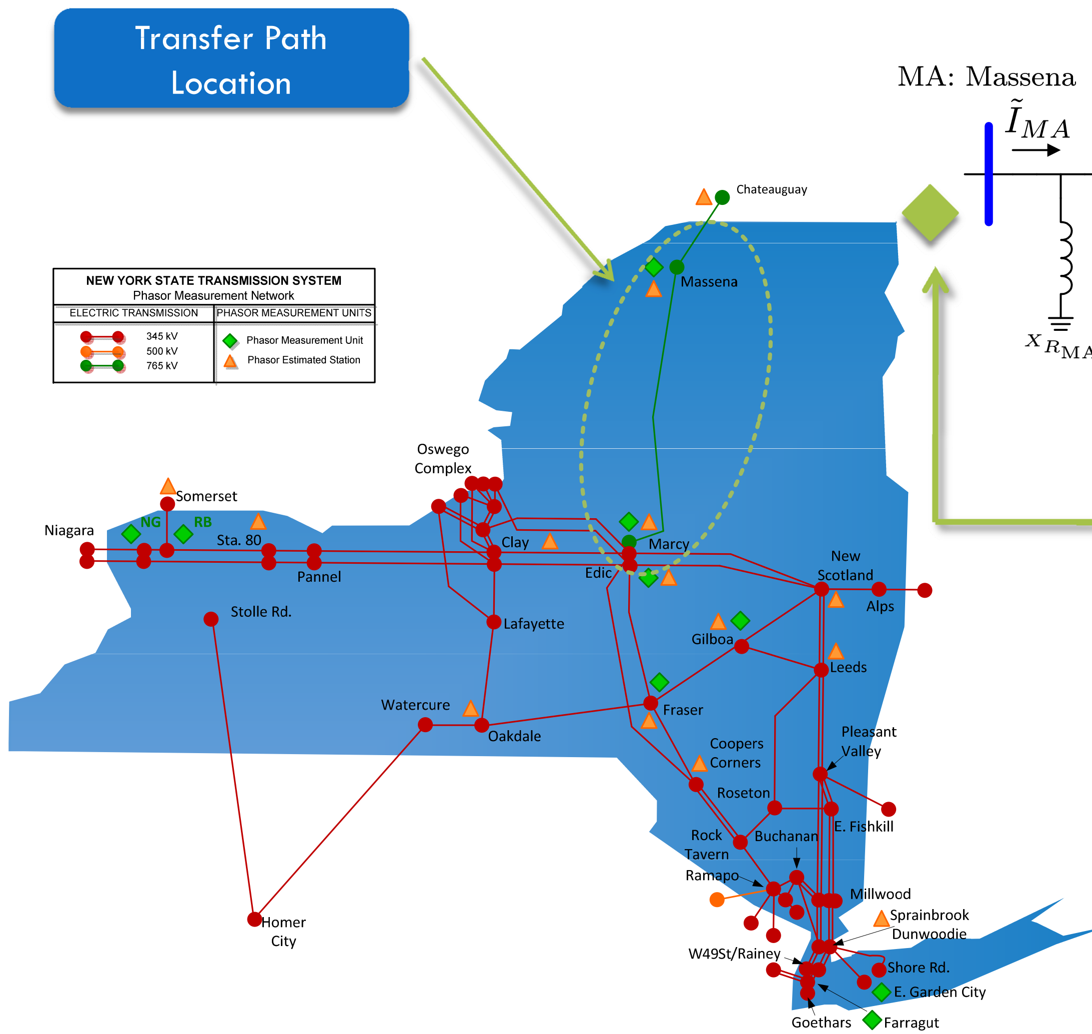
Weighted Nonlinear Least Squares (ii)

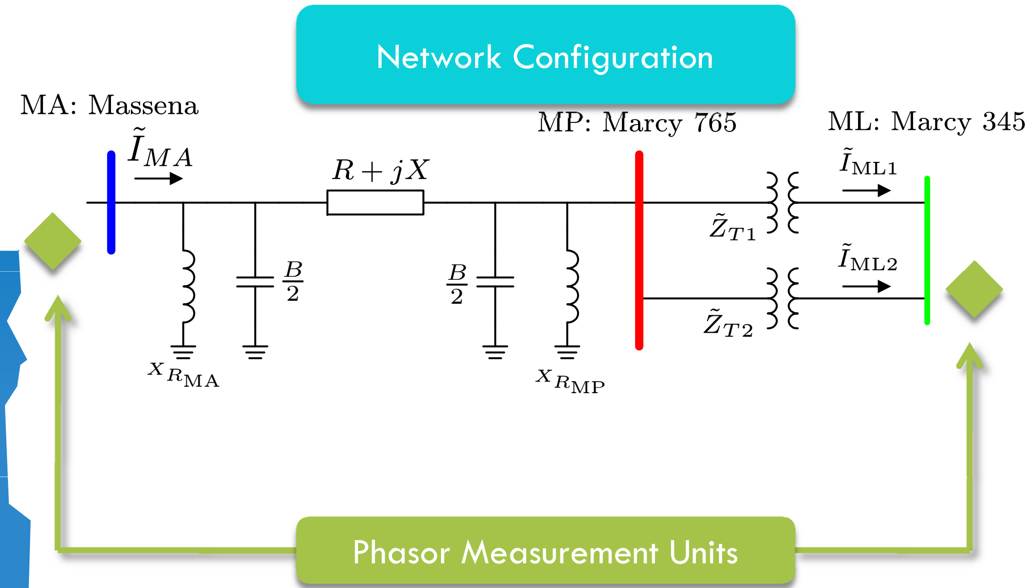
- WLS Solution includes $\mathbf{W}$:
  - State vector update:

$$\mathbf{s}^{(k)} = -\mathbf{H}\left(\mathbf{W}\,\mathbf{J}\left(\mathbf{x}^{(k)}\right)\right)^{T}\mathbf{W}\,\mathbf{f}\left(\mathbf{x}^{(k)}\right)$$

- $\mathbf{W}$ does not change at each iteration.
- It is updated for each set of measurements, and is computed from PMU Data for each set *n* of synchrophasors (a "snapshot").

NY Marcy-Massena Transfer Path (i)

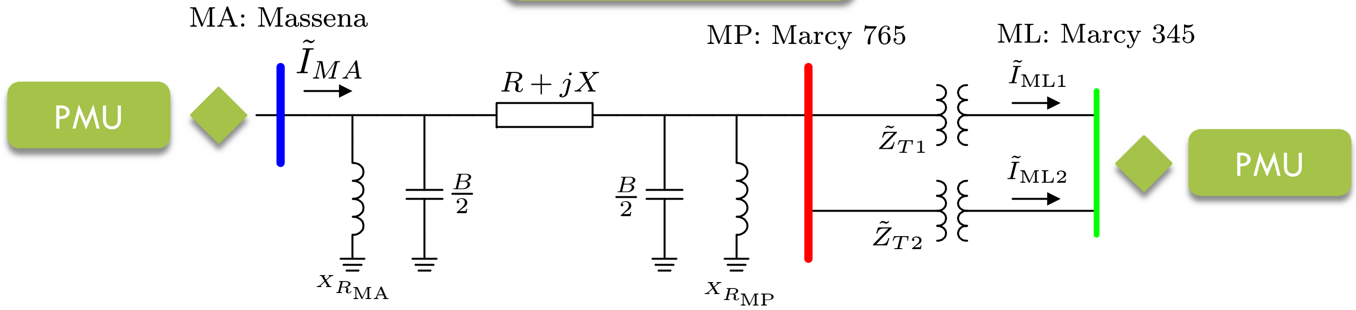




- We use this transfer path to illustrate the ideas of the Phasor State Estimator concept.

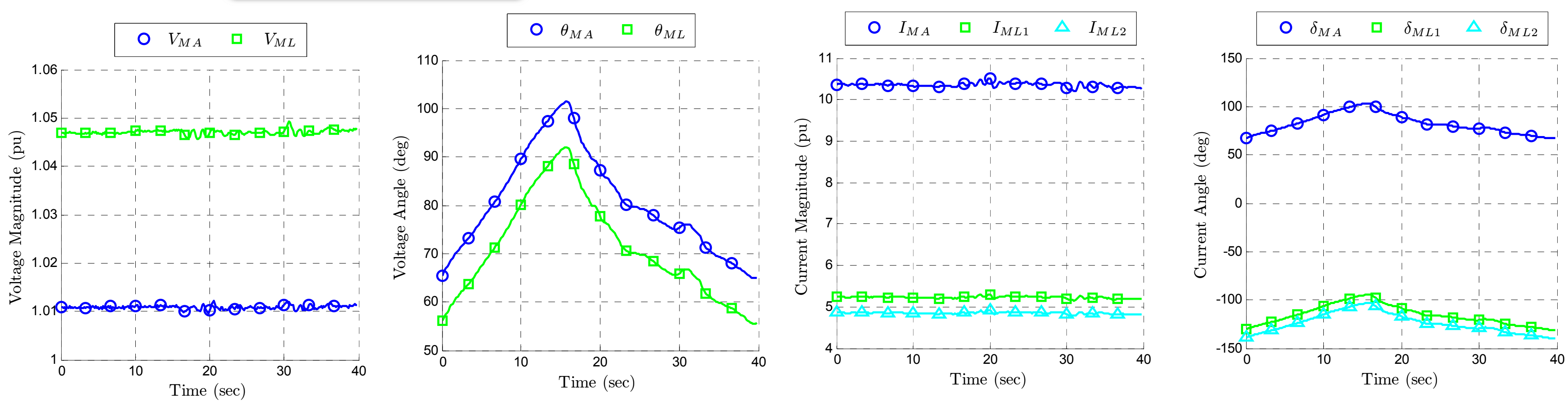
NY Marcy-Massena Transfer Path (ii)

Sample PMU Data



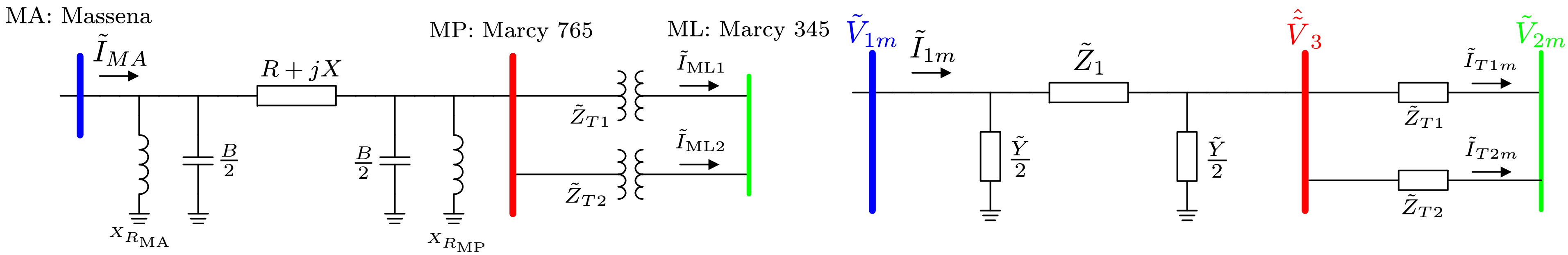
Voltage Magnitude and Angle

Current Magnitude and Angle



Application to the ML-MA Transfer Path

- Marcy-Massena Transfer Path Equivalent Circuit



- Build a model containing the network and the measurement data.
- Network Model:
  - Calculate $\hat{\tilde{V}}_3$ from $\tilde{V}_1$ and $\tilde{V}_2$:

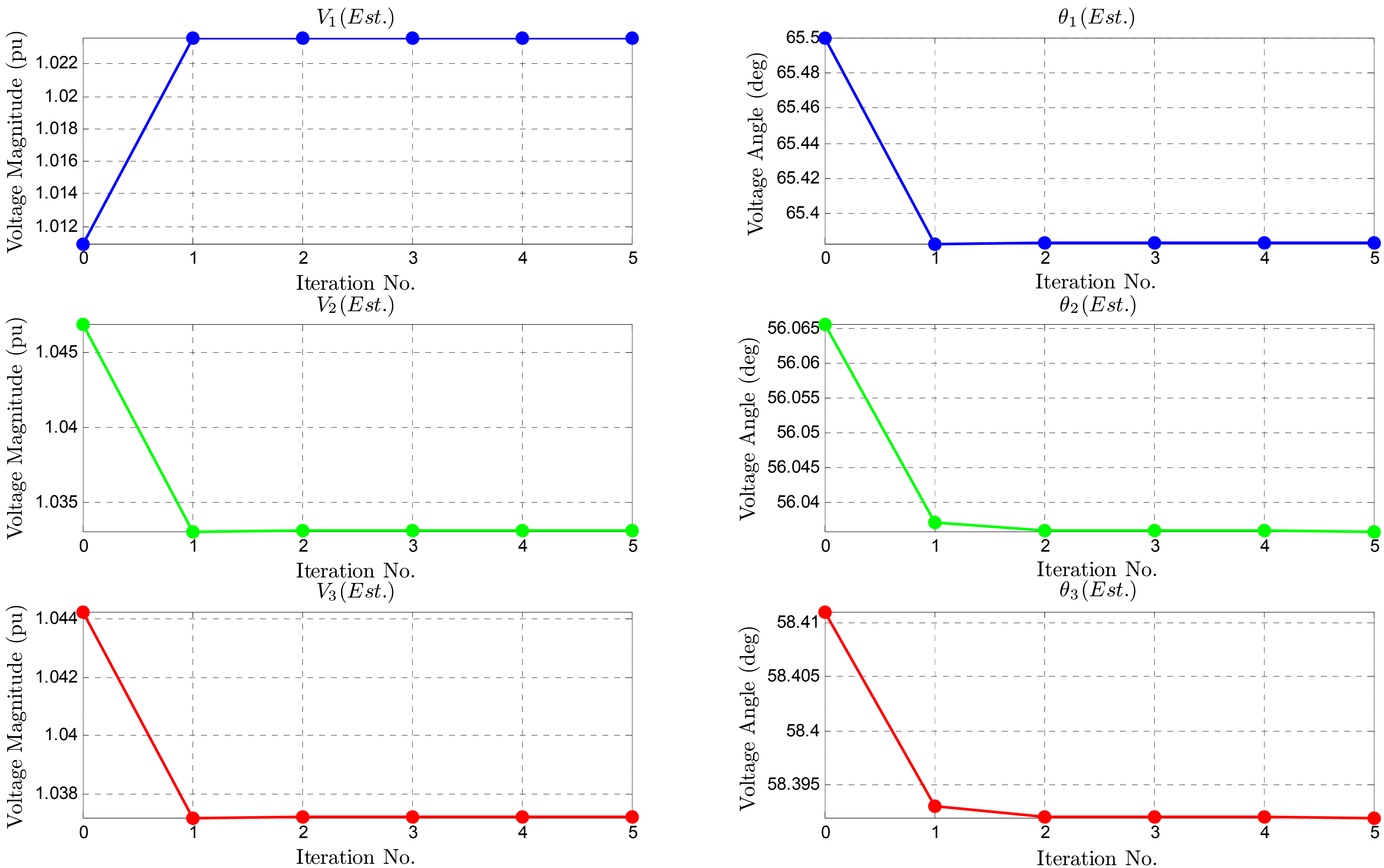
LHS: $$\hat{\tilde{V}}_3 = \underbrace{\left(1 + \tilde{Z}_1 \frac{\tilde{Y}}{2}\right)}_{\tilde{Z}_\gamma} \tilde{V}_1 - \tilde{Z}_1 \tilde{I}_1$$

RHS: $$\hat{\tilde{V}}_3 = \tilde{V}_2 + \tilde{Z}_{T1} \tilde{I}_{T1}$$
$$\hat{\tilde{V}}_3 = \tilde{V}_2 + \tilde{Z}_{T2} \tilde{I}_{T2}$$

Performance of the WLS for a Single Measurement

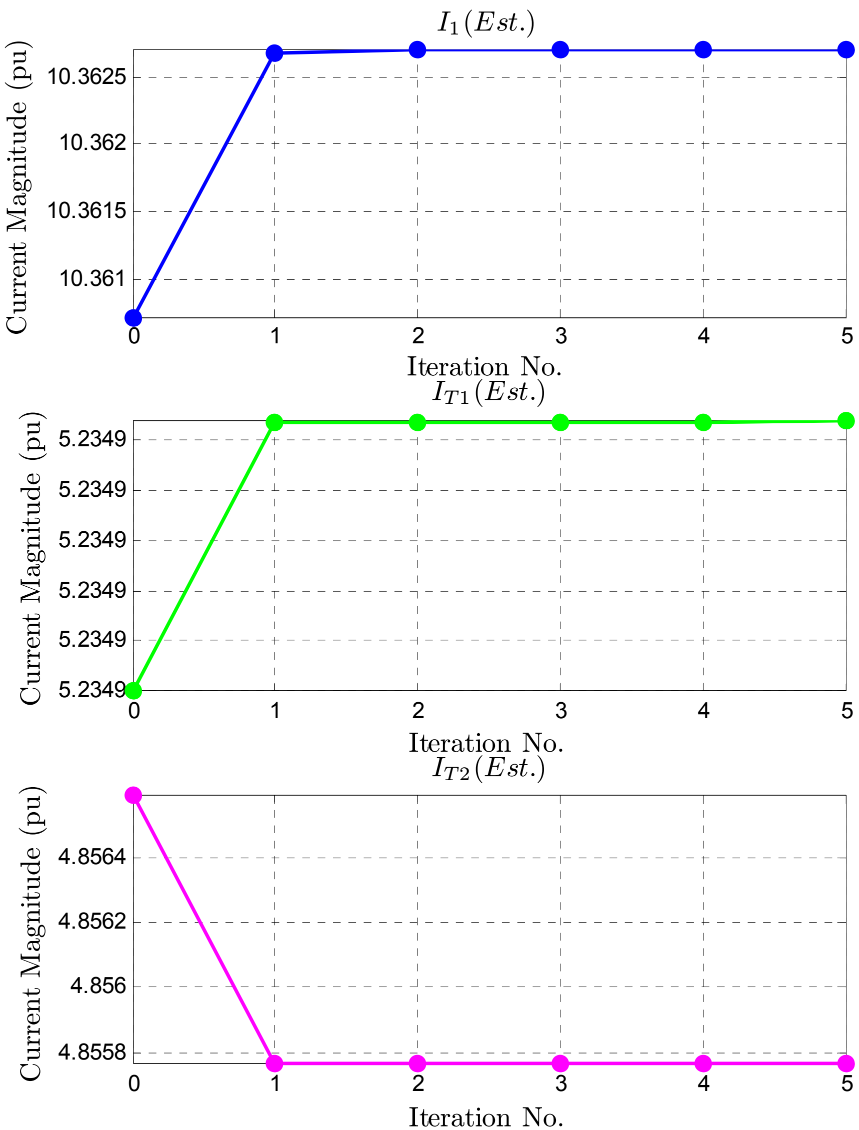
Voltage Magnitude Convergence

Voltage Angle Convergence

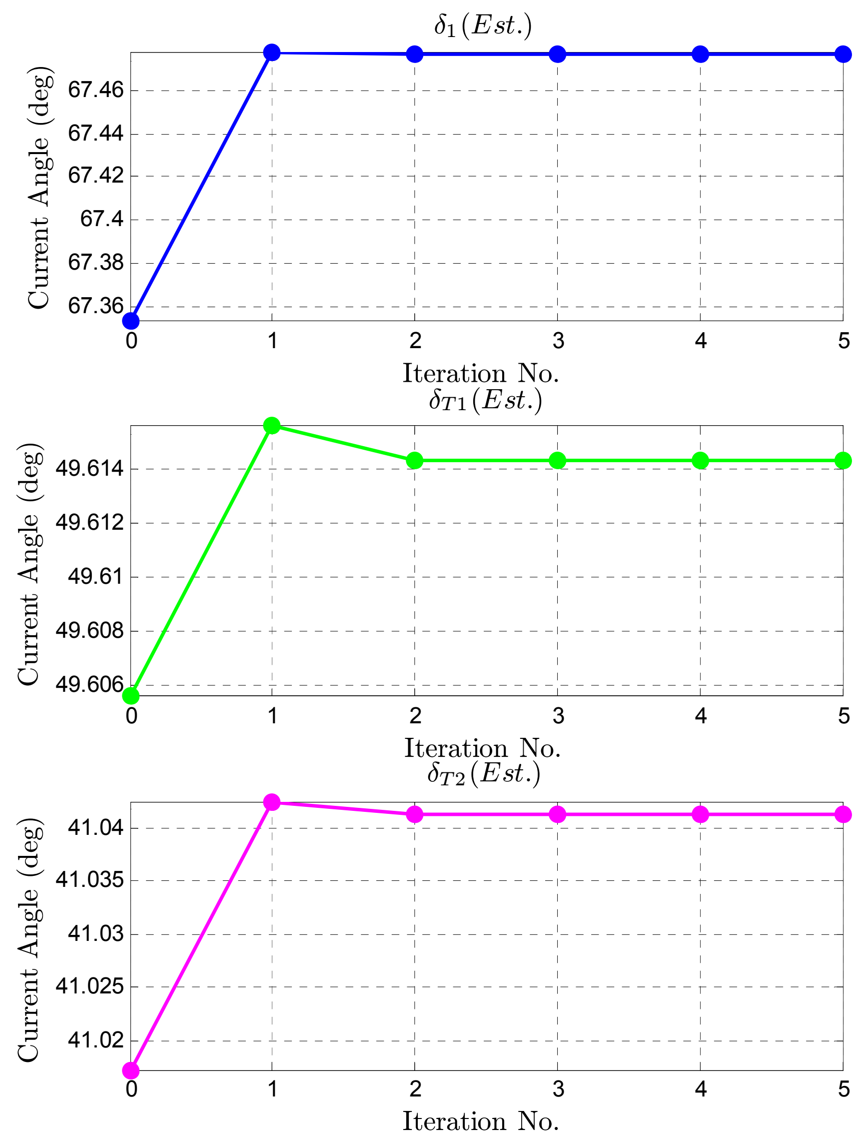


Performance of the WLS for a Single Measurement

Current Magnitude Convergence

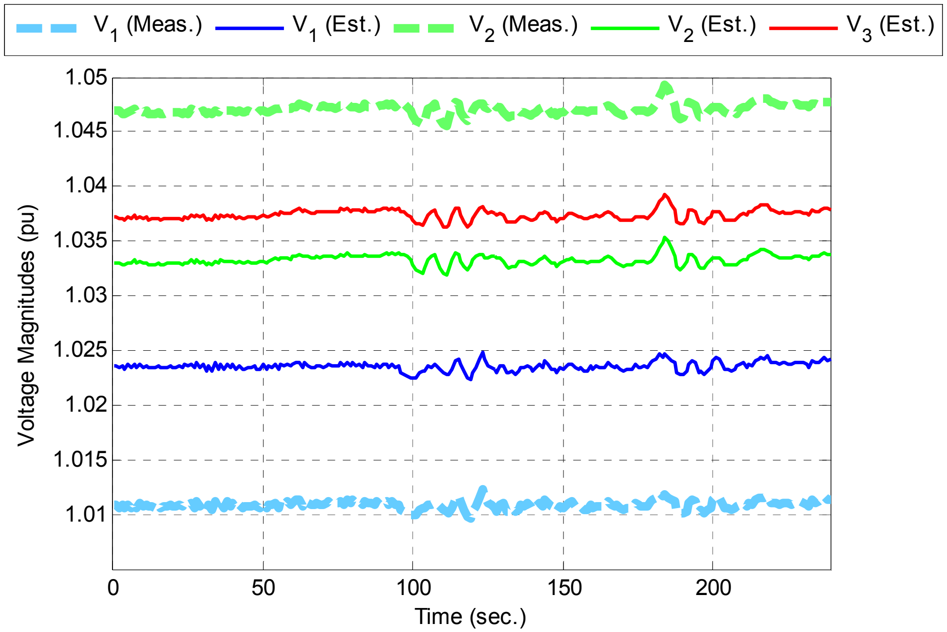


Current Angle Convergence

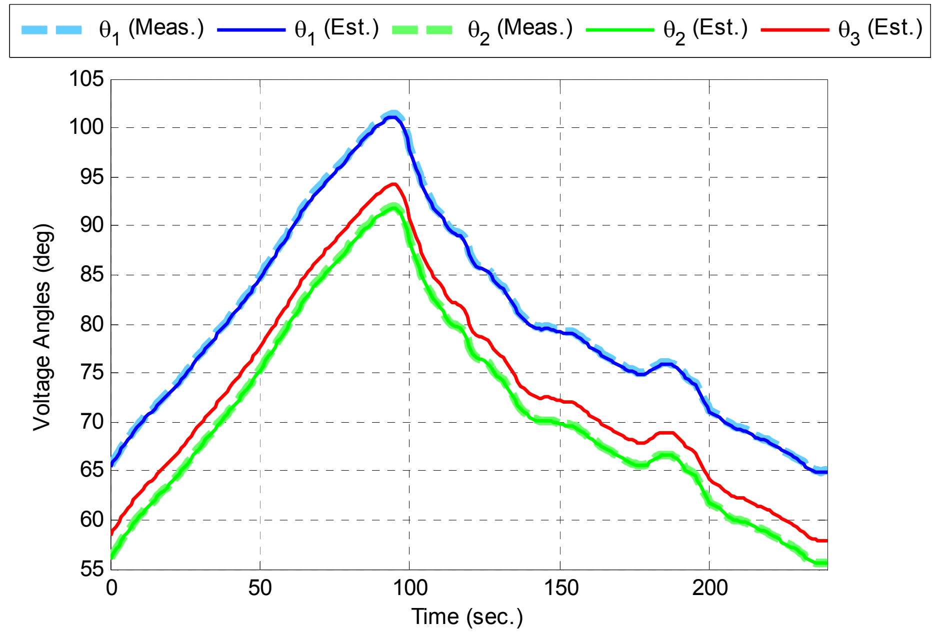


Performance of the WLS for a Set of Synchrophasor Measurements

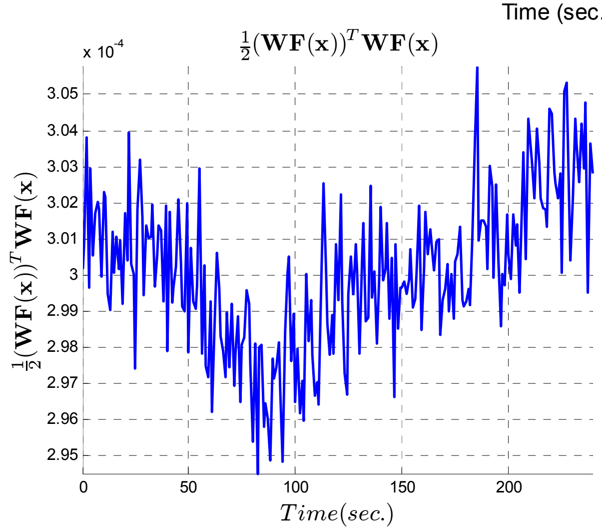
Estimated and Measured Voltage Magnitudes



Estimated and Measured Voltage Angles

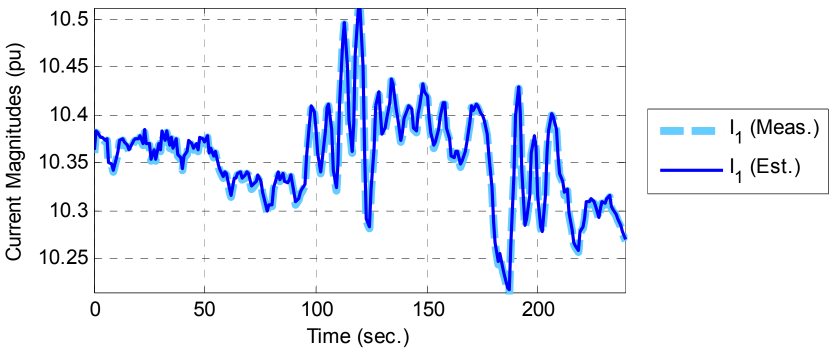


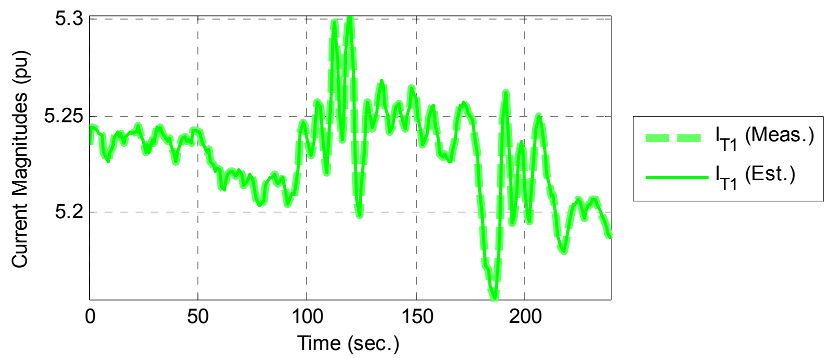
Sum of Squares Error

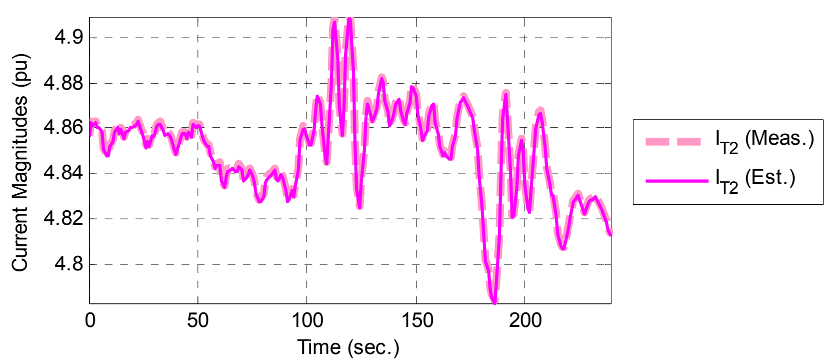


Performance of the WLS for a Set of Synchrophasor Measurements

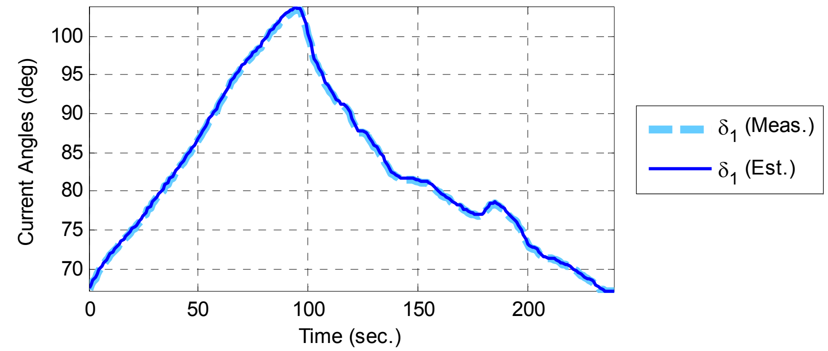
Estimated and Measured Current Magnitudes

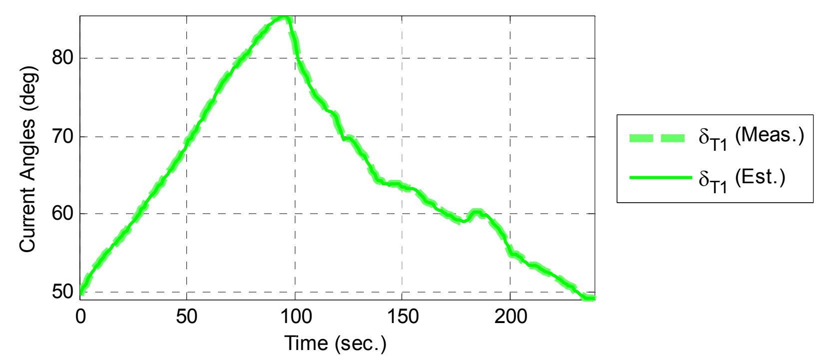


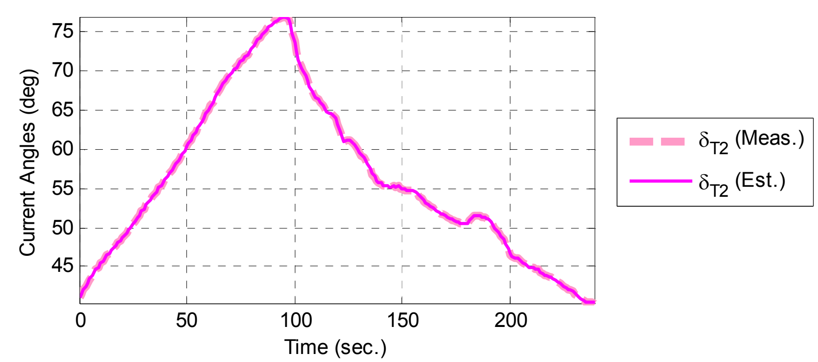




Estimated and Measured Current Angles



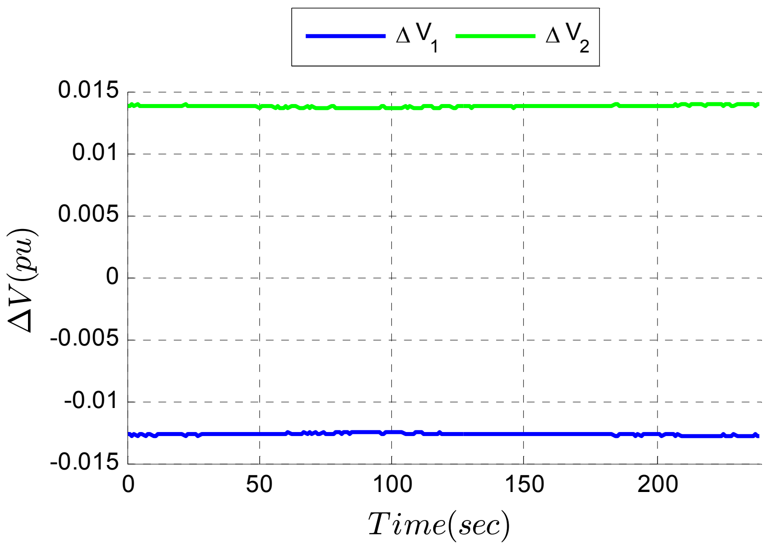




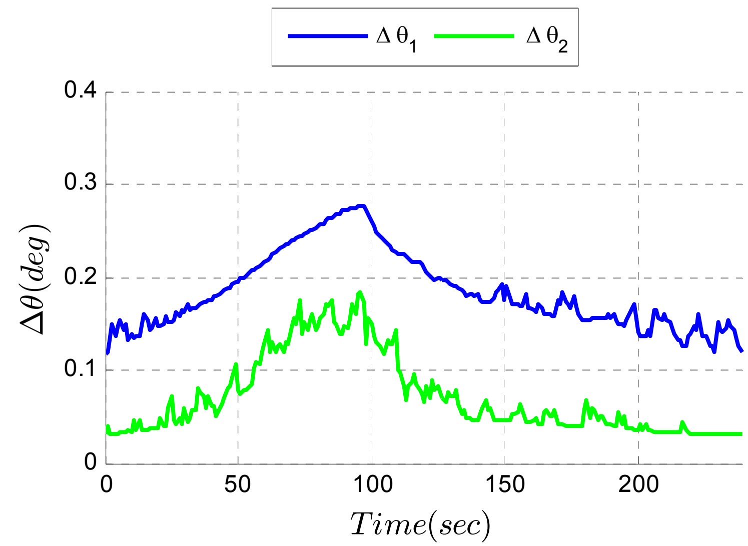
Performance of the WLS for a Set of Synchrophasor Measurements

Difference Between the WLS Estimate and Measurement of the System States: $\Delta x_i = \hat{x}_i - x_{im}$

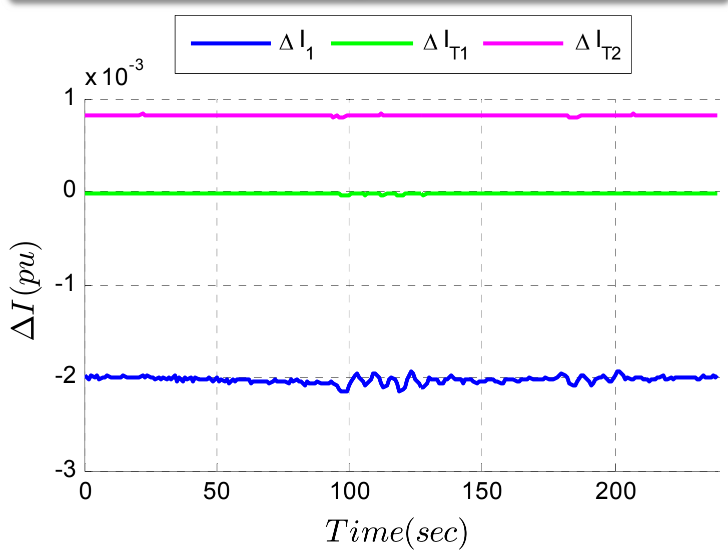
Voltage Magnitude Error



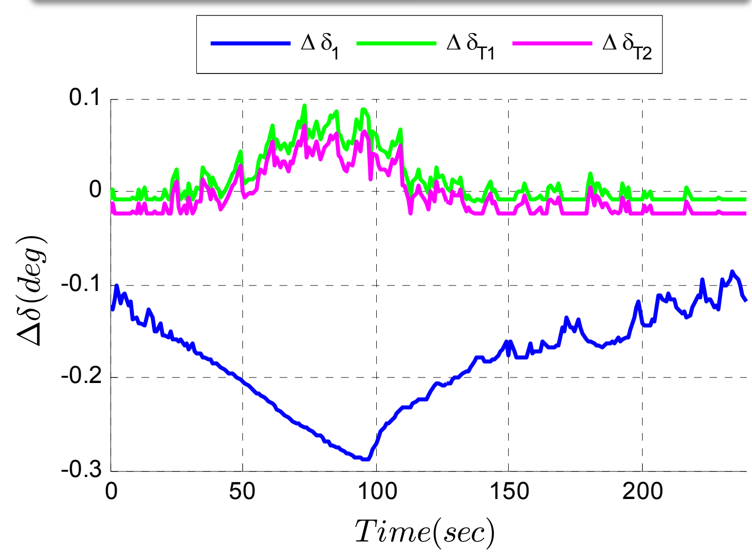
Voltage Angle Errors



Current Magnitude Errors

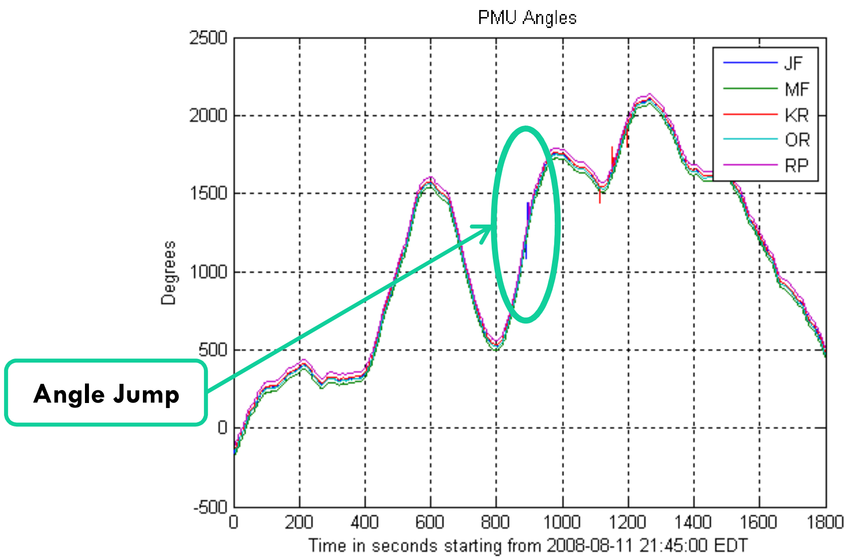


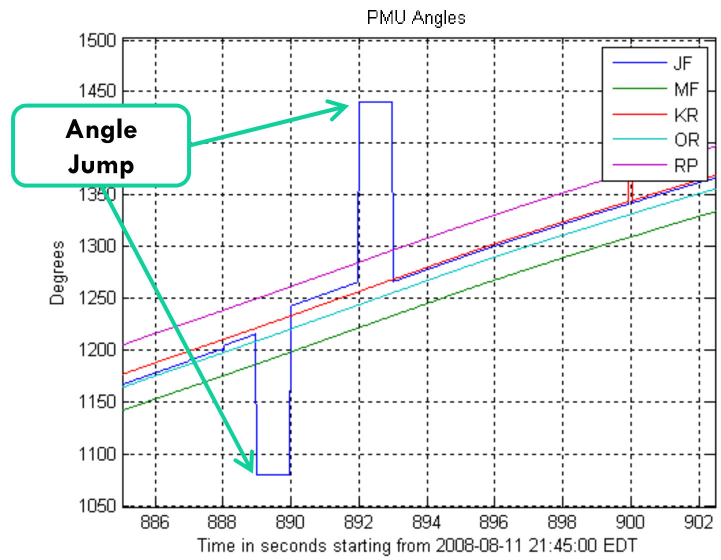
Current Angle Errors



Angle-Bias Errors on PDC PMU Data

- What kind of angle-bias can the PSE correct?
  - Angle-biases due to communication delays
  - And the implementation of the Bus angle calculation of each manufacturer.
- Observed Angle-Bias in AEP





Angle-Bias Detection and Correction

- Extension to the PSE Model
  - Modify the **angle** measurement equations as follows:
    - For each PMU-Bus $i \neq 1$

$$f^*_{i_\theta} : \theta_i - \theta_{im} + \Omega_{si} = 0$$

Modified Voltage Angle Measurement Equations

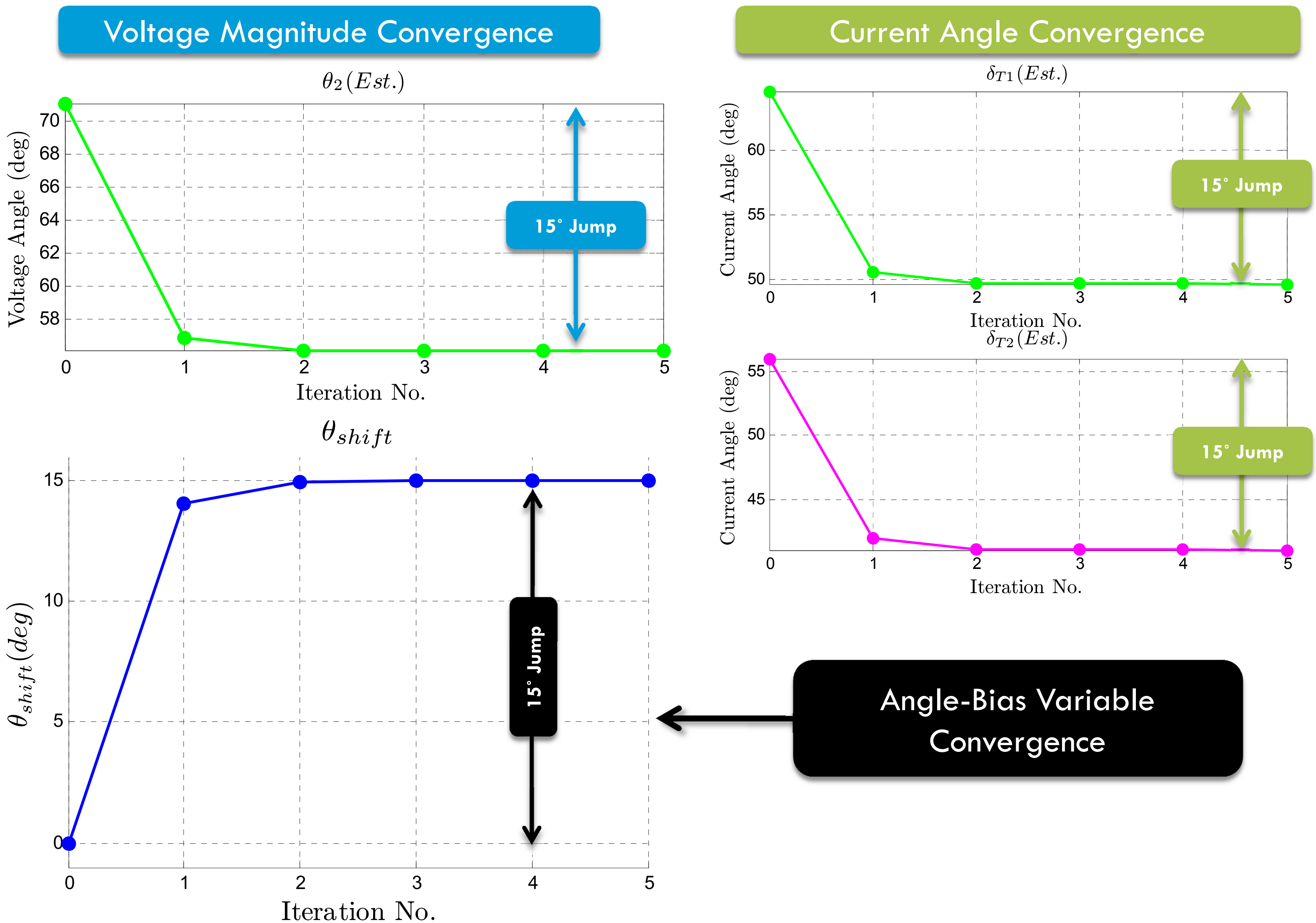
$$f^*_{i_{k\delta}} : \delta_{i_k} - \delta_{i_k m} + \Omega_{si} = 0$$

Modified Current Angle Measurement Equations

  - The new Jacobian elements have to be accounted for.
  - Achievable if the Angle-Bias Condition is satisfied.
    - Total number of equations is greater or equal to the total number of unknowns including angle-bias variables:

$$N_T \geq U_T$$

Performance of the WLS for a Single Measurement – Case for an Angle Shift of 15° on Bus 2



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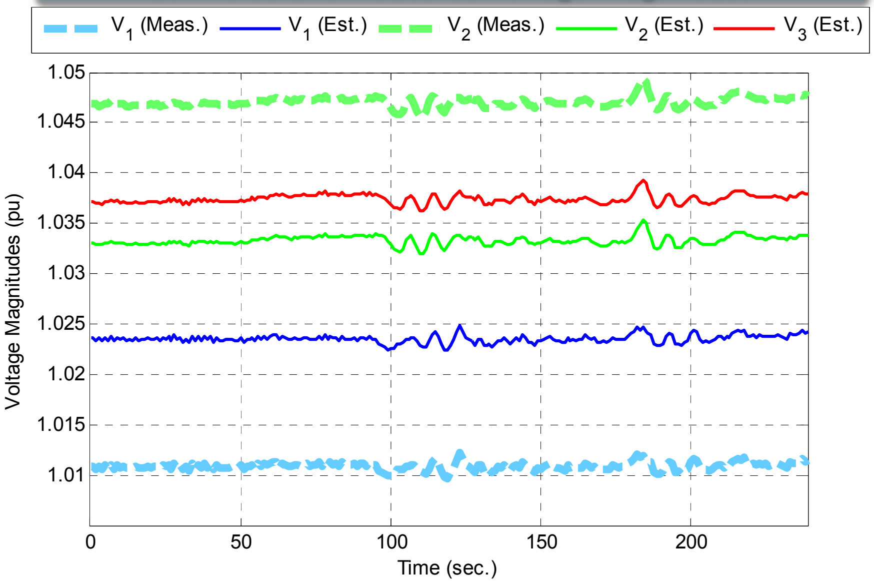
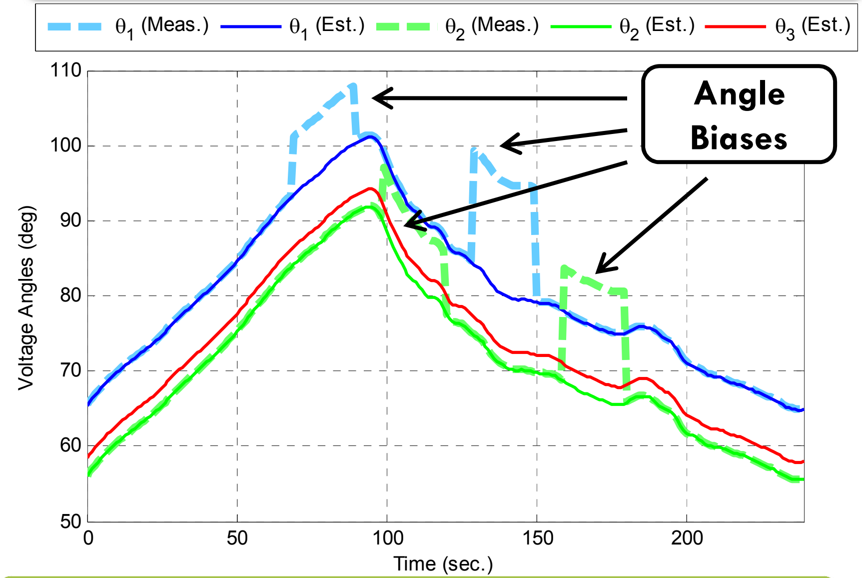


Estimated and Measured Voltage Angles



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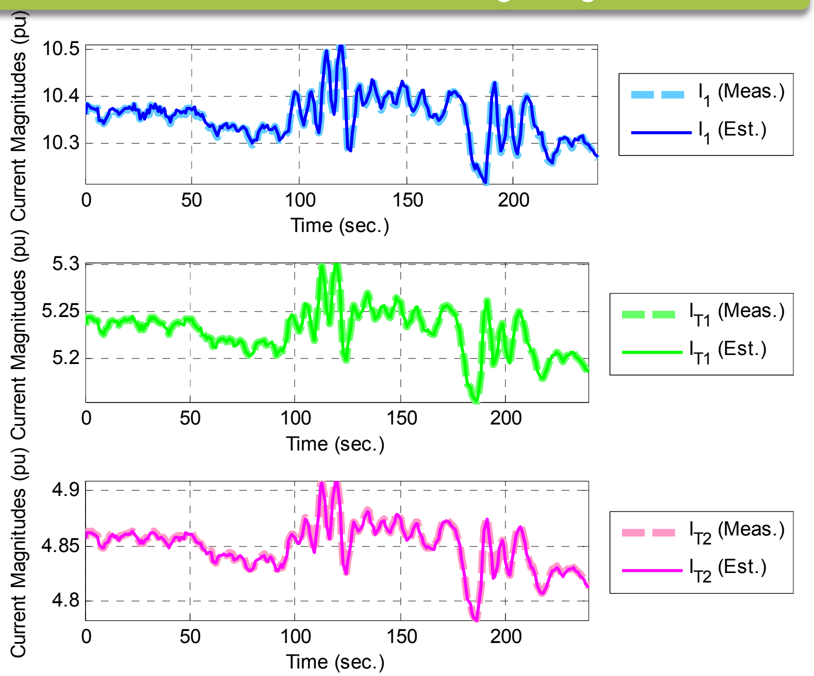
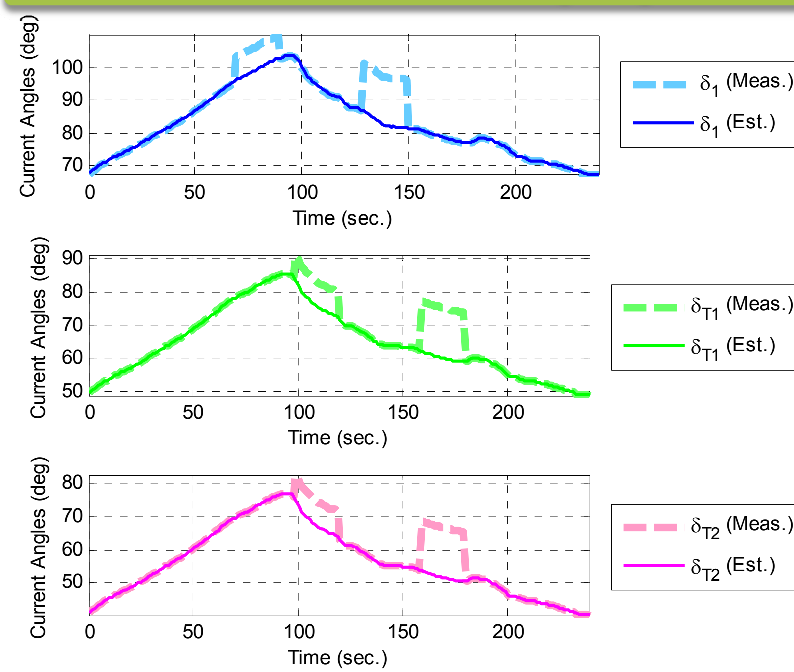
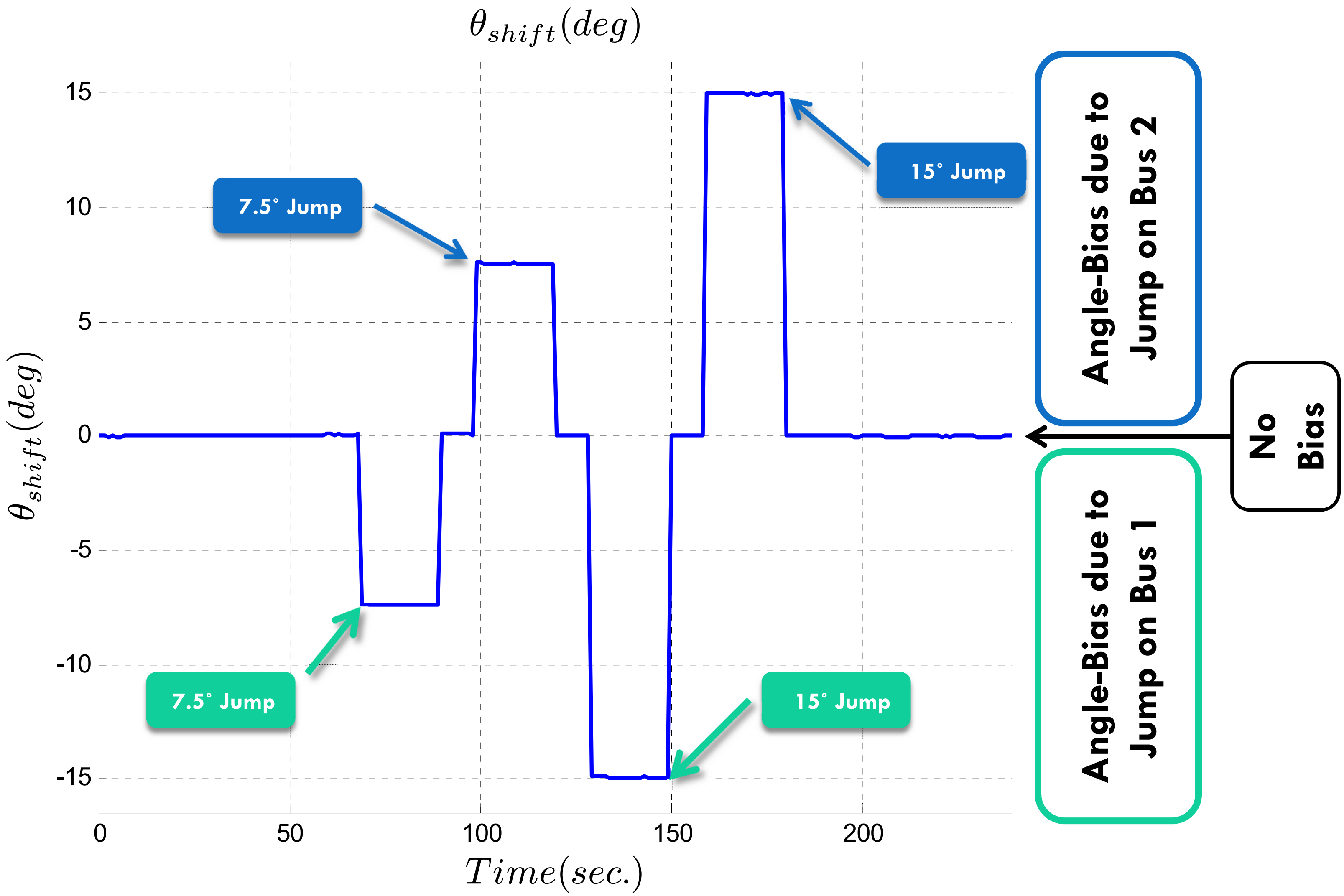


Estimated and Measured Voltage Angles



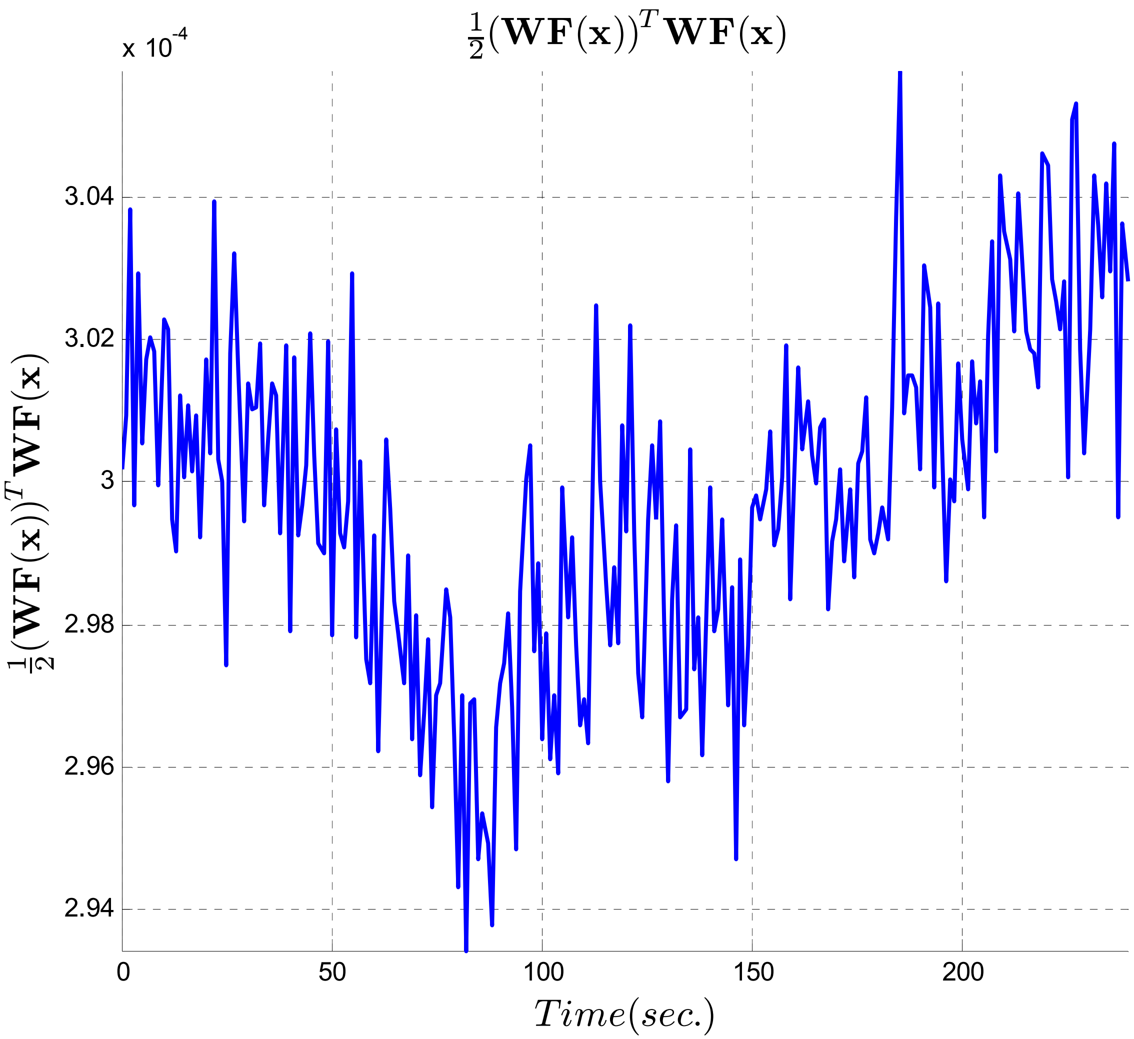
Performance of the WLS for a Set of Synchrophasor Measurements

Angle-Bias Detection



Performance of the WLS for a Set of Synchrophasor Measurements

Sum of Squares Error



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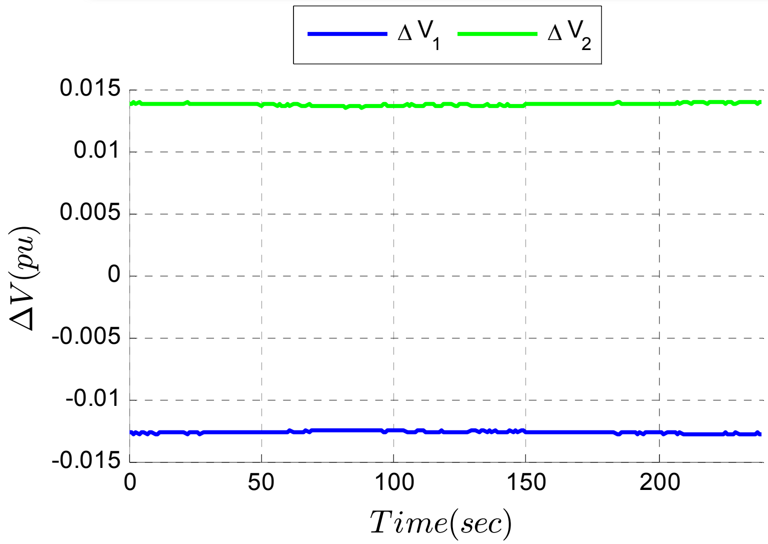
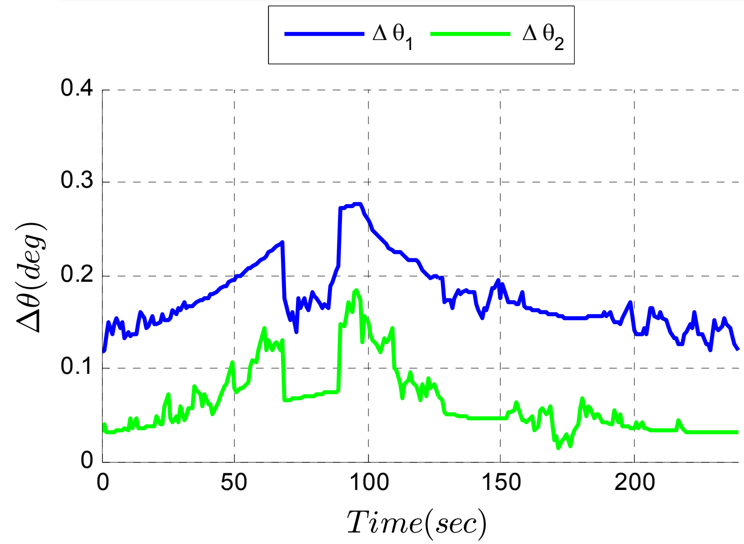
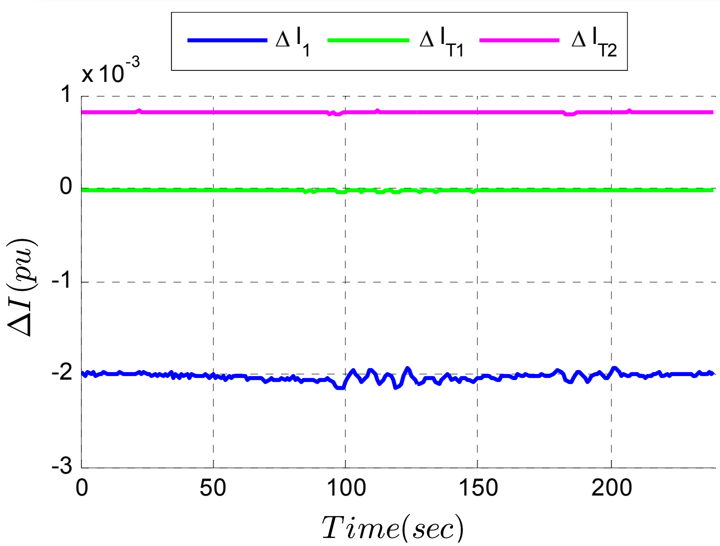


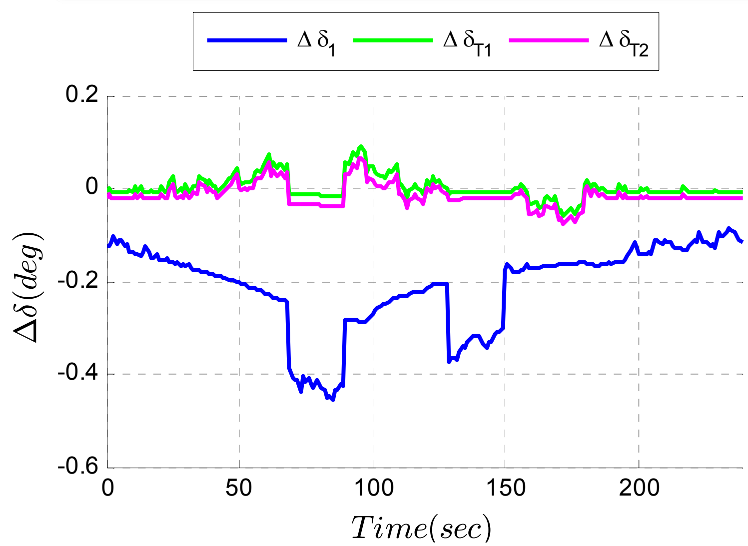
Voltage Angle Errors



Current Magnitude Errors



Current Angle Errors



On-going and Further Work

- Observability Analysis
  - H Matrix Conditions
    - PMUs needed for full observability
    - PMUs needed for full angle-bias observability
    - PMUs needed for PSE with Calibration
  - Observability of Islanded Sections of the HV Network
- Phasor State Estimation with Calibration
  - Taking windows of data to calibrate scaling factors of PTs and CTs

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