Measurement Based, Non-Iterative Direct State Calculation for Power Networks

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1. Formulate Equations From Measurements

In our method, the measurement equations (bus voltage magnitudes, line PQ flows, and bus PQ injections) are formulated in rectangular coordinates of the bus voltages. With this formulation, the nonlinear measurement equations become quadratic polynomial equations of the voltage variables. We keep track of the voltage components that make up each quadratic variable.

Voltage Magnitude

For bus i:

$$V_{iR}^2 + V_{iI}^2 = V_{iM}^2$$

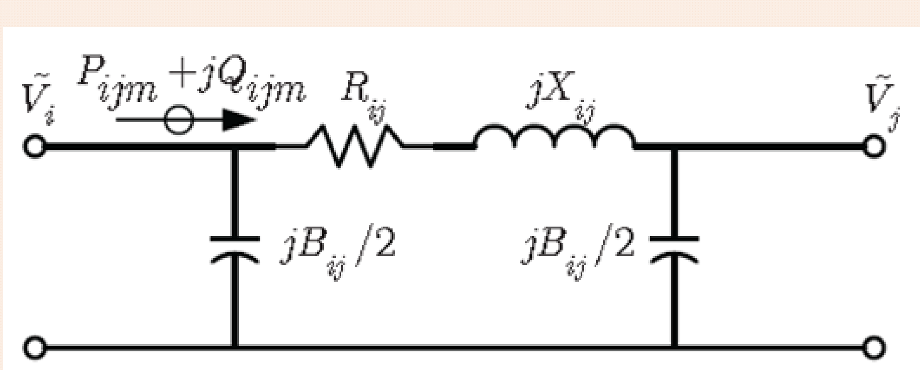
Line Power Flow

For line flow from bus i to j:

$$g_{ij}(V_{iR}^2 + V_{iI}^2 - V_{iR}V_{jR} - V_{iI}V_{jI}) + h_{ij}(V_{iI}V_{jR} - V_{iR}V_{jI}) = P_{ijm}$$

$$h_{ij}(V_{iR}^2 + V_{iI}^2 - V_{iR}V_{jR} - V_{iI}V_{jI}) + g_{ij}(V_{iR}V_{jI} - V_{iI}V_{jR}) - \frac{B_{ij}}{2}(V_{iR}^2 + V_{iI}^2) = Q_{ijm}$$

$$g_{ij} = \frac{R_{ij}}{Z_{ij}^2}, h_{ij} = \frac{X_{ij}}{Z_{ij}^2}, Z_{ij}^2 = R_{ij}^2 + X_{ij}^2$$



Bus Injection

For bus i, add all line flow equations to and from the bus together.

All Together

In matrix form:

$$A_\xi \xi = c$$

Where A_ξ is the coefficients matrix, ξ is the unknown quadratic variables and c is the measurement values.

Introduction

This poster describes a new method for solving for the states of a nonlinear AC power system in a non-iterative manner when given an adequate set of sufficiently accurate measurements from the system. This method is based on the Kipnis-Shamir relinearization technique which is used to solve over-defined sets of polynomial equations. This new state calculation method provides the same results as traditional iterative state estimation methods, and the method does not require an initial guess of system states. PMU measurements can be added to speed up the calculation.

2. Split into Linearly Independent and Dependent Parts

Reformulate the system into:

$$Ay + Bz = c$$

Where A is the linearly independent columns of A_ξ and B is the rest of the system. y and z are parts of the quadratic variable vector ξ that correspond to A and B. We then solve for y in terms of c and z in a least squares formulation.

$$y = d + Dz$$

$$d = (A^T A)^{-1} A^T c, D = -(A^T A)^{-1} A^T B$$

3. Apply Relinearization Technique

Because there is usually more measurements than what is needed for observability, we can apply the Kipnis-Shamir relinearization technique. In the relinearization technique, we expand the system to a set of linear equations in a higher variable space for the vector t. t is composed of the z terms and the quadratic combinations of the y and z terms. One equation can be generated for t if:

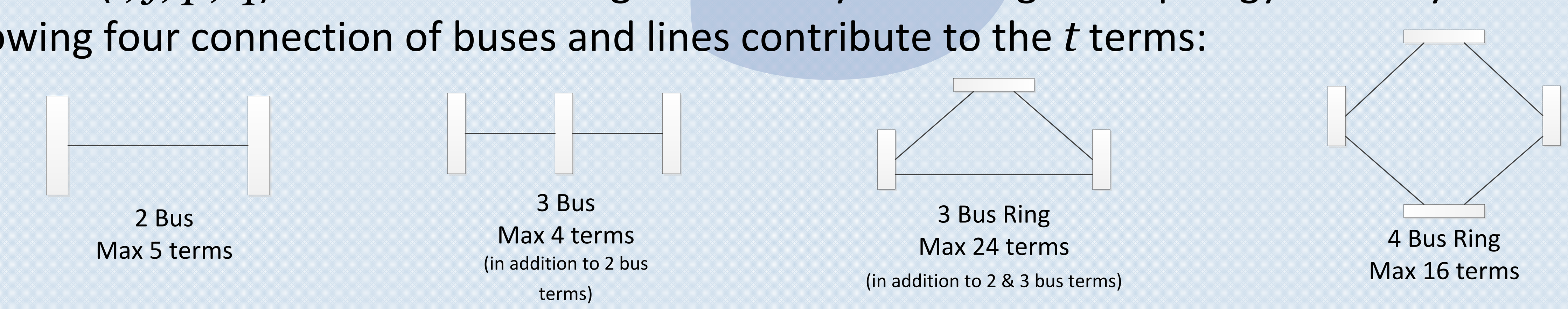
$$y_{ij} y_{pq} = (x_i x_j)(x_p x_q) = (x_i x_p)(x_j x_q) = y_{ip} y_{iq}$$

Where x are the individual voltage components and these quadratic combinations of ij, pq, ip and jq are present in y and z. The resulting equation for t is:

$$y_{ij} y_{pq} - y_{ip} y_{iq} = y_{l1} y_{l2} - y_{l3} y_{l4} = (d_{l1} + D_{l1} z)^T (d_{l2} + D_{l2} z) - (d_{l3} + D_{l3} z)^T (d_{l4} + D_{l4} z) = 0$$

Where d_{l1} is the $l1^{th}$ entry into the vector d and D_{l1} is the $l1^{th}$ row of the matrix D.

The valid (i, j, p, q) combinations are generated by searching the topology of the system. Only the following four connection of buses and lines contribute to the t terms:



4. Solve for t, y, z

Formulate equations of t into matrix form and remove the zero columns of the matrix:

$$A_t t = k$$

A_t (after zero columns removed) must be non-singular for t to be uniquely determined.

The first N_z entries of t are the values of z (N_z being the number of z terms), and y can be solved with:

$$y = d + Dz$$

5. Extract the voltage components

Now that we have the values of y and z, we can use the indices that we kept to extract the values of the real and imaginary bus voltage components. We first extract the squared voltage terms and use these terms to solve any unsolved components using division.

Incorporating PMU Measurements

PMU measurements can be used to speed up the calculation because they provide the real and imaginary voltage components at the bus. This allows us to eliminate some quadratic terms from the system and also turn other quadratic terms into linear terms which speeds up the solution.

PMUs can also be used to supply the line flow measurements used by the direct state calculation.

Progress and Future Work

Our direct state calculation method has been tested on systems of 140, 300 and 1200 buses to yield correct solutions.

Items being investigated include:

- Effect of measurement noise/error on the solution
- Adding PMU measurements
- Scalability of the method
- Incorporating parallel computing

Acknowledgements

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