PMU-Based Monitoring of Power System Dynamics

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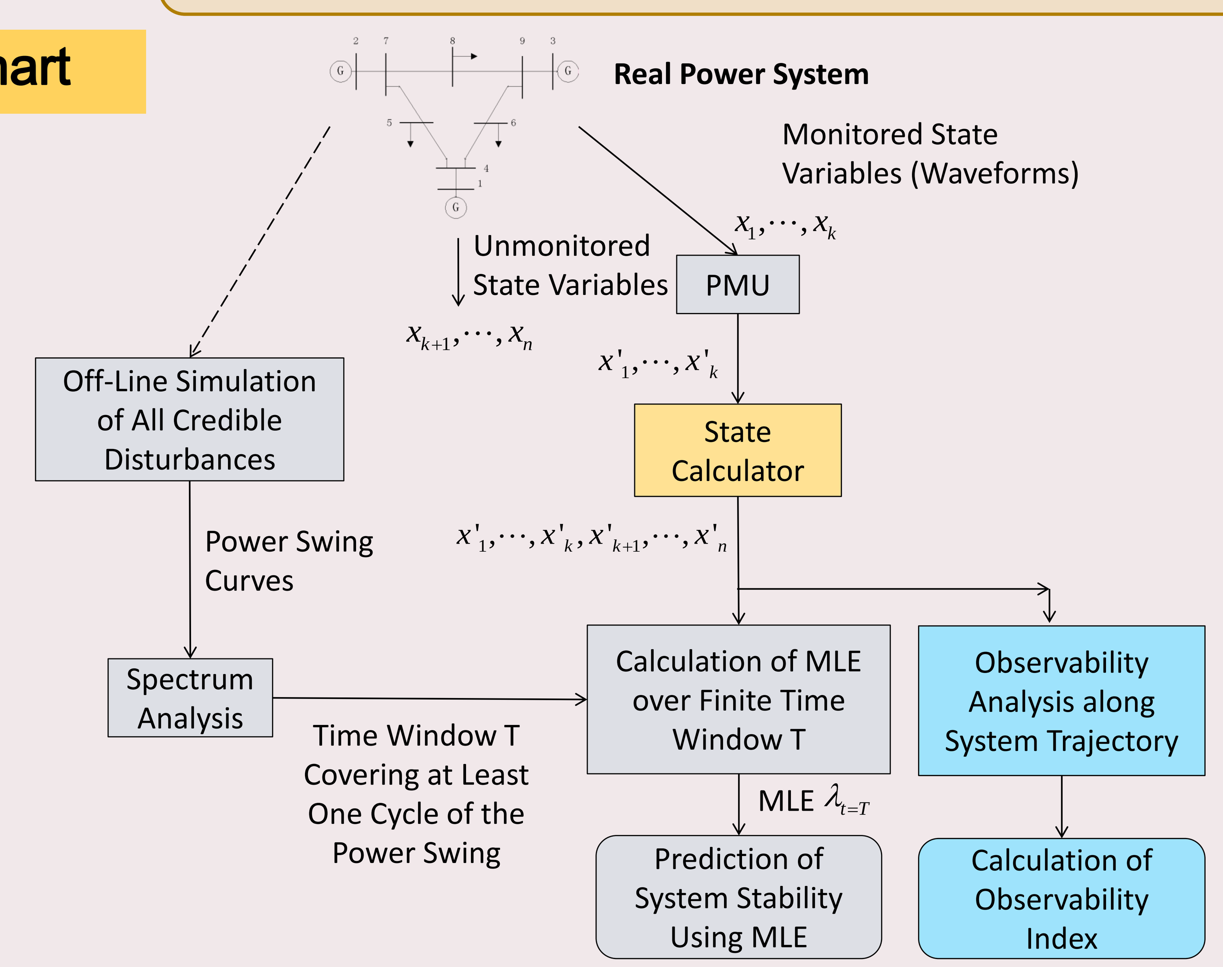
PMU-Based Power System Dynamics Monitoring

Goal Wide area monitoring of rotor angle stability to prevent widespread blackouts.

Methodology
- State Calculator (SC).
- Maximal Lyapunov Exponent (MLE).
- Observability Analysis of Nonlinear Dynamic Systems.

Flowchart



State Calculator and MLE

Dynamic Model of a Power System

$$\dot{x} = f(x) \Rightarrow \begin{cases} \dot{\delta}_i = \omega_i \\ \dot{\omega}_i = h_i(\delta, \omega) \end{cases} \quad i = 1, 2, \ldots, n$$

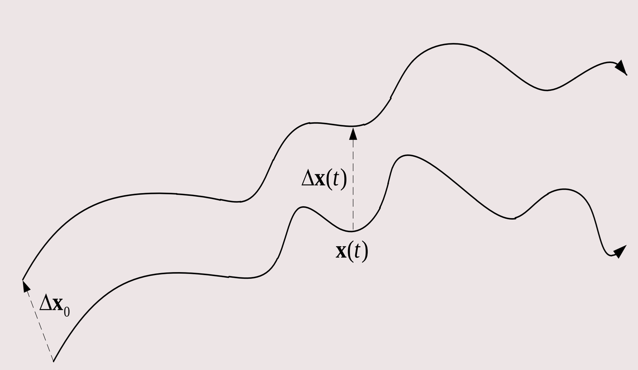
State Calculator

- Assume $x_1(t), \cdots, x_k(t)$ are observed by PMU measurements.
- $x_{k+1}(t), \cdots, x_n(t)$ are estimated by

$$\bar{x}_j(t+\Delta t) = x_j(t) + f(x(t))\Delta t$$
$$x_j(t+\Delta t) = x_j(t) + \Delta t \left\{ \frac{1}{2} f(x(t)) + \frac{1}{2} f(\bar{x}(t+\Delta t)) \right\}, \quad j = k+1, \ldots, n$$
$$x_1(t+\Delta t) = x_1^{observed}(t+\Delta t)$$
$$\vdots$$
$$x_k(t+\Delta t) = x_k^{observed}(t+\Delta t)$$

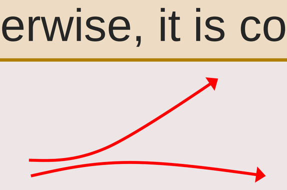
MLE

- The Lyapunov Exponents are used to characterize the exponential divergence or convergence of nearby trajectories.



- MLE is calculated by the approximation of : $\|\Delta x(t)\| \approx e^{MLE \times t} \|\Delta x_0\|$

- MLE of x(t) is calculated to monitor rotor angle stability after a disturbance.
  —— If MLE<0, the system is (asymptotically) stable.
  —— Otherwise, it is considered "unstable".



Divergent trajectories: Unstable

Converging trajectories: Stable

Nonlinear Dynamic Observability

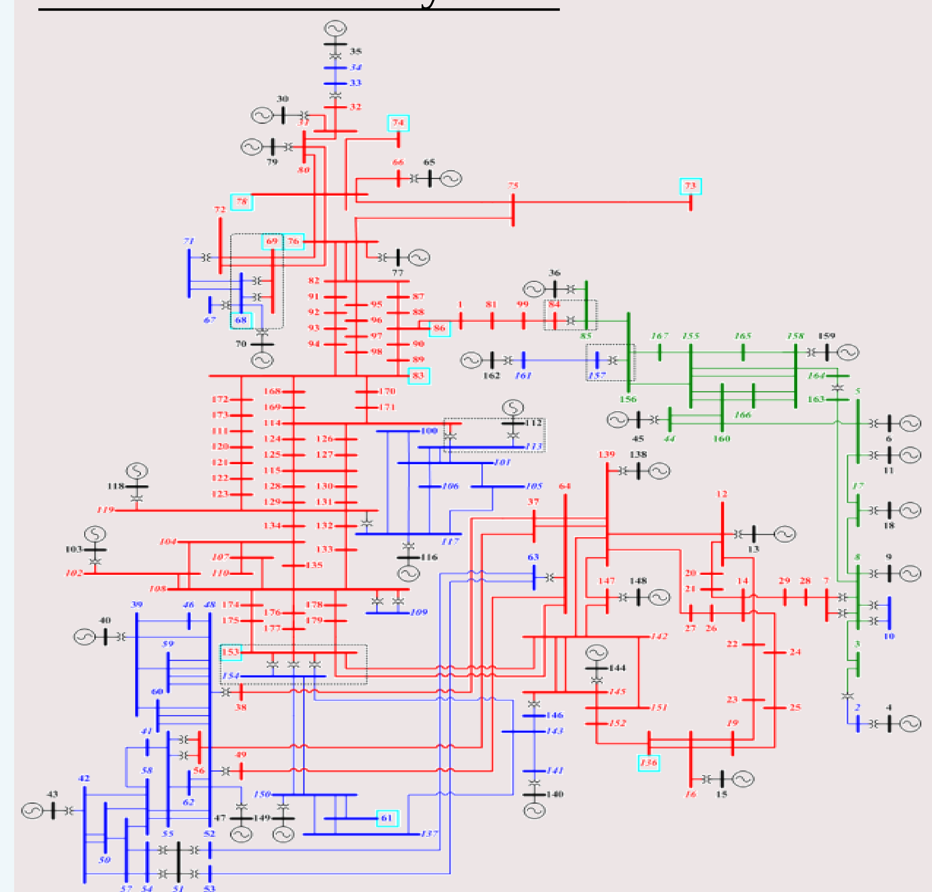
A dynamic system is observable if, for any time t, the current state x(t) can be determined using only the measurements h(x(t)).

- The system is locally observable at x_i ,if the matrix $O(x_i) = \frac{\partial l(x)}{\partial x}\Big|_{x=x_i}$ has a full rank. $O(x)$ is called the observability matrix, in which

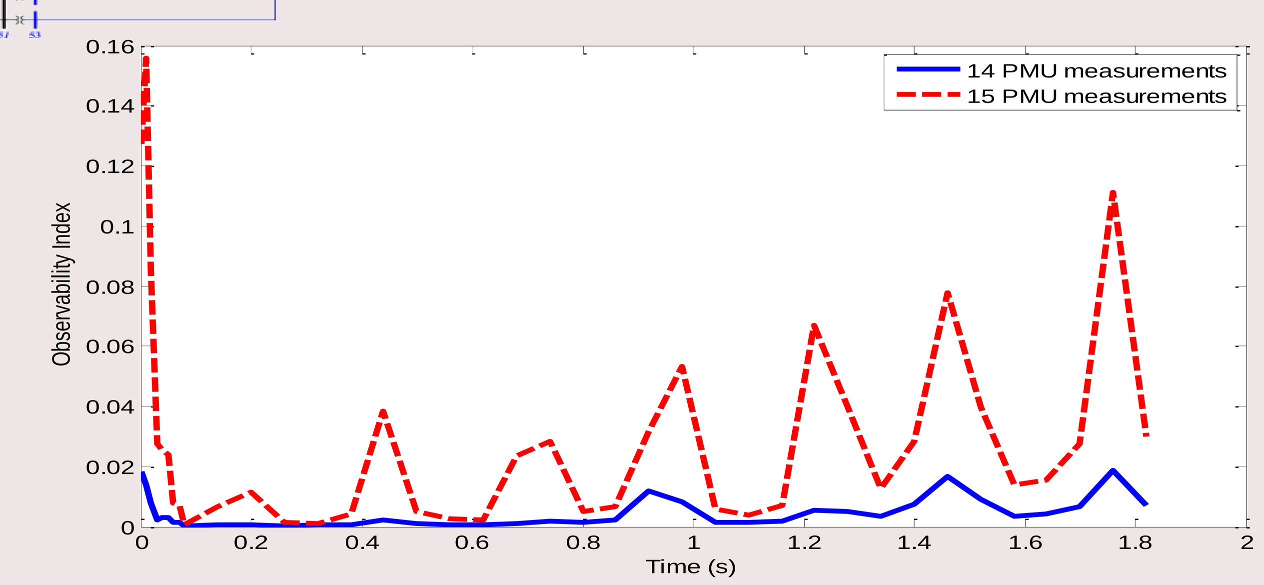
$$l(x_i) = \begin{bmatrix} L_f^0 h_1(x) \\ \cdots \\ L_f^0 h_p(x) \\ L_f^1 h_1(x) \\ \cdots \\ L_f^{n-1} h_p(x) \end{bmatrix}, \quad L_f^0 h = h, \quad L_f^1 h = L_f h = \begin{bmatrix} \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} & \cdots & \frac{\partial h}{\partial x_n} \end{bmatrix} \begin{bmatrix} f_1(x) \\ f_2(x) \\ \cdots \\ f_n(x) \end{bmatrix}, \quad L_f^2 h = L_f(L_f h), \ldots$$

- The ratio of smallest and largest singular value of observability matrix is chosen as an index to assess the level of system observability.
- Observability indices can be tracked along system trajectory.

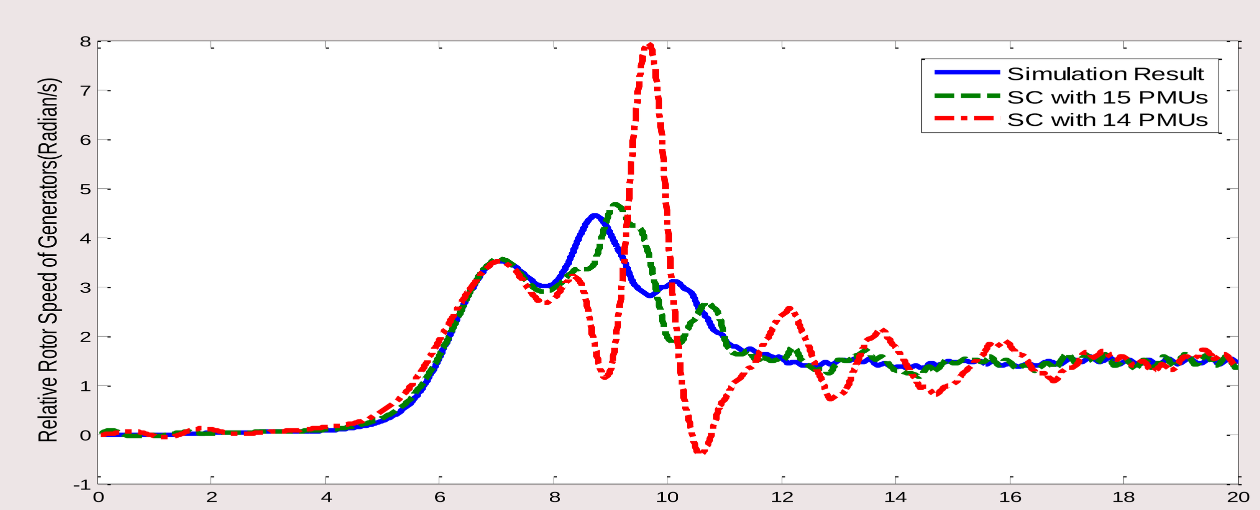
179-Bus WECC System:



- As the number of PMUs increases, the level of observability also improves.



Observability for 14 and 15 PMUs



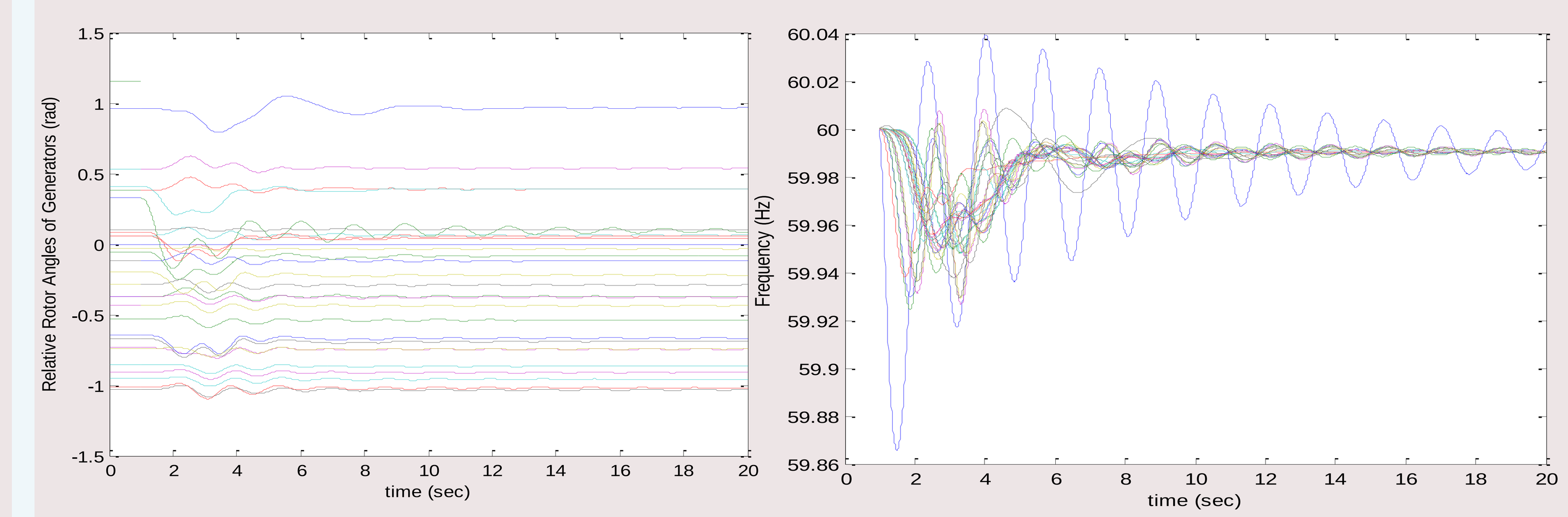
State Calculator with 14 and 15 PMUs

PMU number	Generators with PMUs
14	1,8,9,10,15,16,17,18,19,20,21,22,24,27
15	1,4,8,9,10,15,16,17,18,19,20,21,22,24,27

- The observability indices are much higher and always above 0, but their values differ significantly.
- The observability of 15-PMU scenario is always higher than the 14-PMU scenario.
- The SC results with 15 PMUs are much closer to the simulation results. That indicates that 15 PMUs and for that specific location provide a sufficient level of confidence based on the system observability.

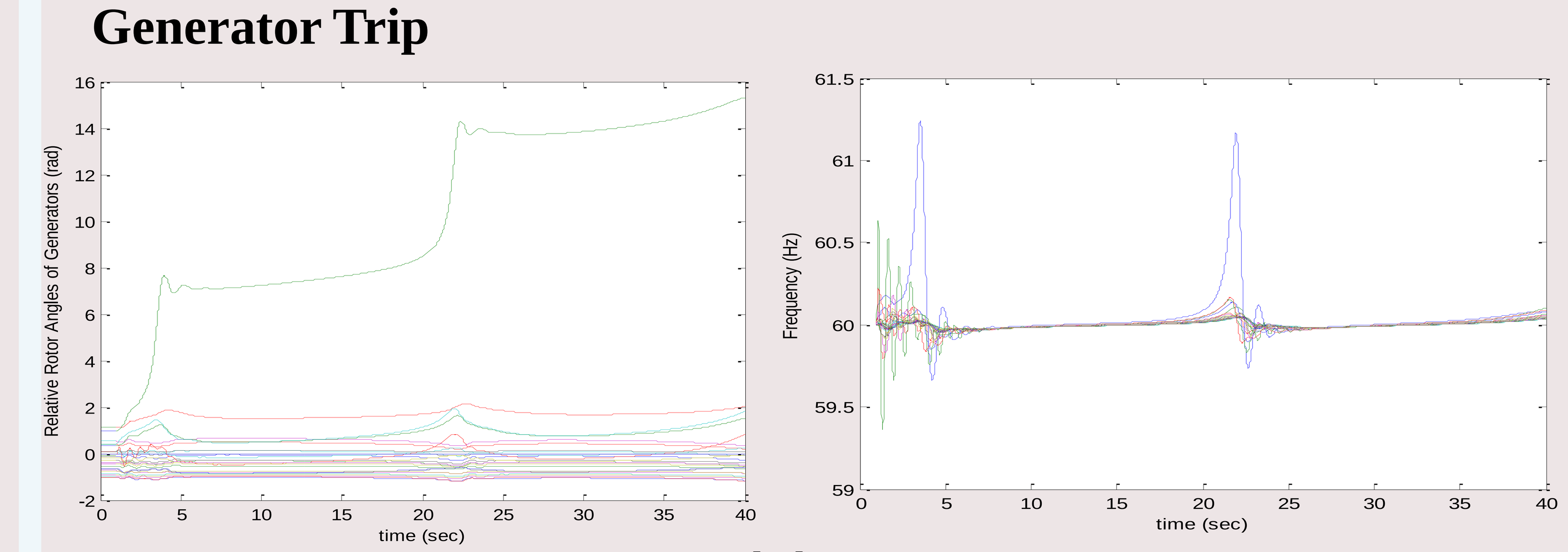
Rotor Angle Stability Analysis Results

Line Trip



MLE = -0.23555 ←→ System is stable

Generator Trip



MLE = 0.72849 ←→ System is unstable

Computational Performance

System Size	179-bus WECC	15246-bus (PJM system)
Generator number	29	1964
State Calculator	0.0495s	3.5918s
MLE	0.0434s	16.5111s

Conclusions

- Developed MLE and State Calculator as effective techniques to predict loss of synchronism in power systems.
- Developed an index to quantify the level of observability for different PMU deployment scenarios.

NASPI, March 2015, Burlingame, CA