

# **Developing Methods for Oscillation Source Location by Extracting Interharmonic Components from Synchrophasor Data**

Presentation by  
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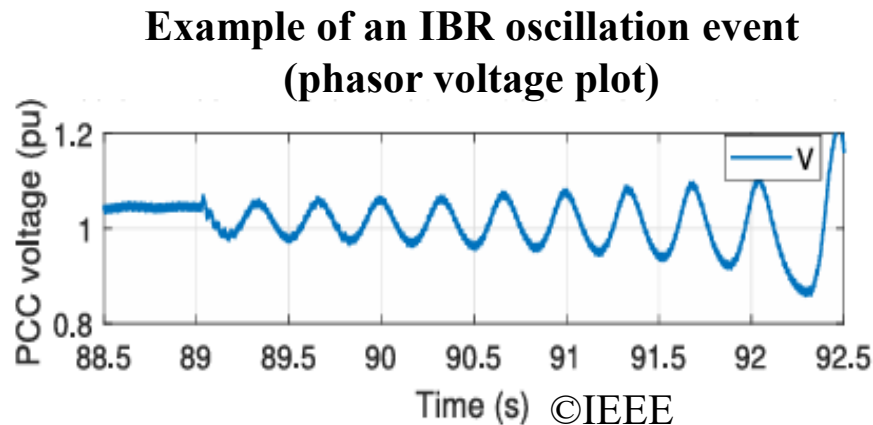
- 1. Review: Phasor oscillations are caused by interharmonics**
- 2. How to extract interharmonics from phasor data?**
- 3. Case studies and sensitivity analysis**
- 4. Conclusions and takeaways**

# **1. Review:**

## **Phasor oscillations are caused by interharmonics**

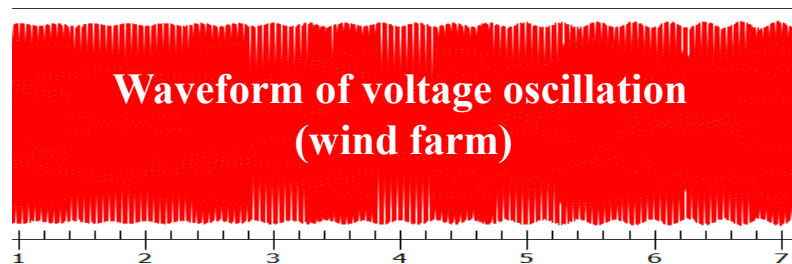
# 1A: The phenomena of power system oscillation

- Power system oscillation is a system stability concern over the years;
- There is a renewed interest in oscillations due to
  - 1) IBR interconnections
  - 2) increased monitoring capabilities such as PMUs
- Almost all monitoring activities and stability studies are based on **phasor representations** of the phenomena
- For example, efforts have been made to use PMU data to locate sources causing oscillations



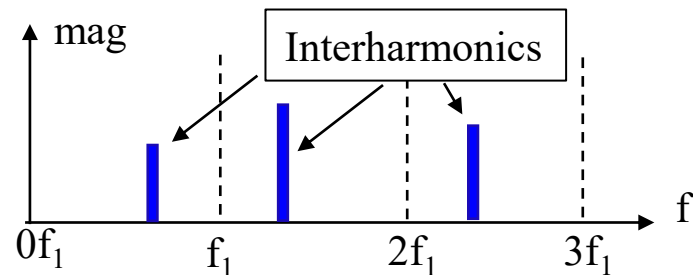
# 1B: Oscillations seen from the waveform perspective

Have you ever wondered what the waveform underlying an oscillating phasor looks like?



- It exhibits a beating wave pattern => Let's do spectral analysis
- There are spectral components called interharmonics, which cause the beating pattern

IEC 61000-4-30 definition: Interharmonics (IHs) are spectral components which reside between harmonic frequencies. Interharmonics here refers to both spectral components below and above fundamental frequency  $f_1$

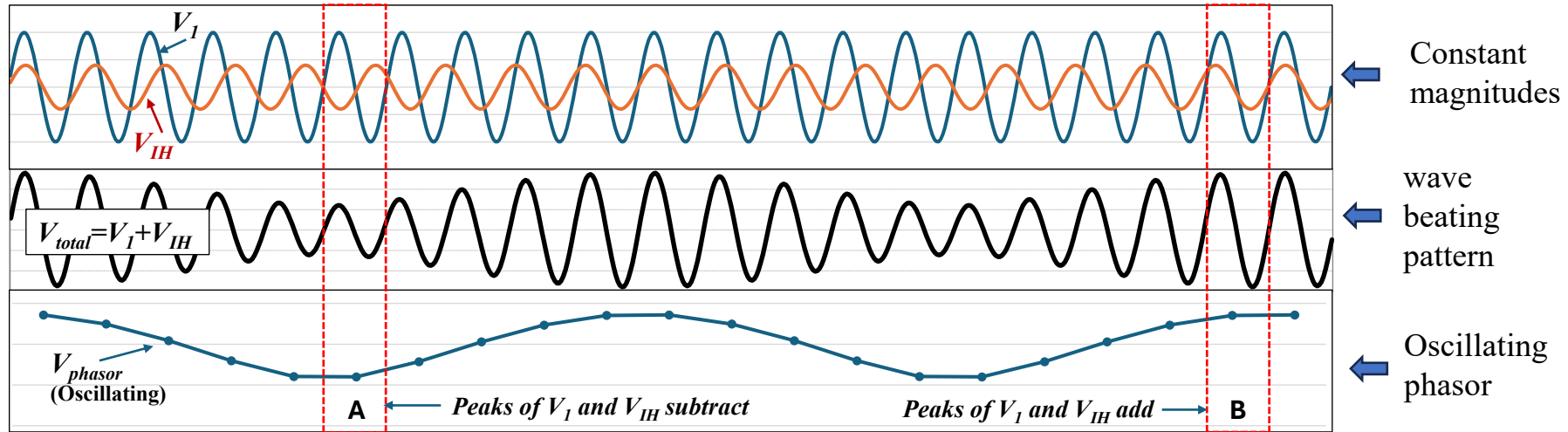


# 1C: Why are phasor oscillations caused by interharmonics?

## Existence of interharmonic components is the necessary and sufficient condition of phasor oscillation

A 60Hz wave containing one IH:  $v_{total}(t) = v_1(t) + v_{IH}(t) = \sin(2\pi f_1 t) + 0.3 \times \sin(2\pi f_{IH} t + \theta_{IH})$

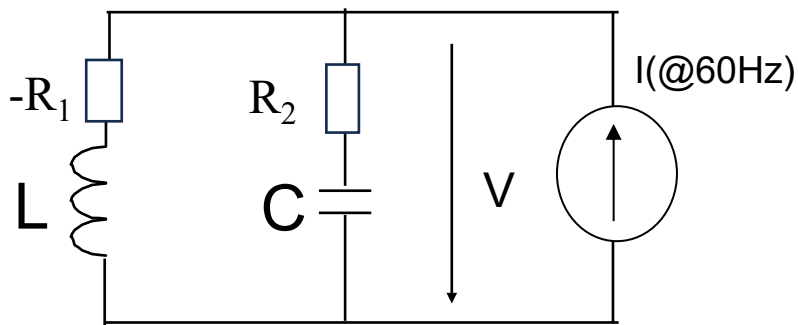
PMU definition of (measured) 60Hz phasor:  $\vec{V}_{phasor}(k) = \frac{\sqrt{2}}{T_1} \int_{(k-1)T_1}^{kT_1} w(t)v(t)e^{-j2\pi f_1 t} dt \Rightarrow |\vec{V}_{phasor}(k)| \approx RMS$



Note: Proof of the necessary condition can be found from a paper shown at the end

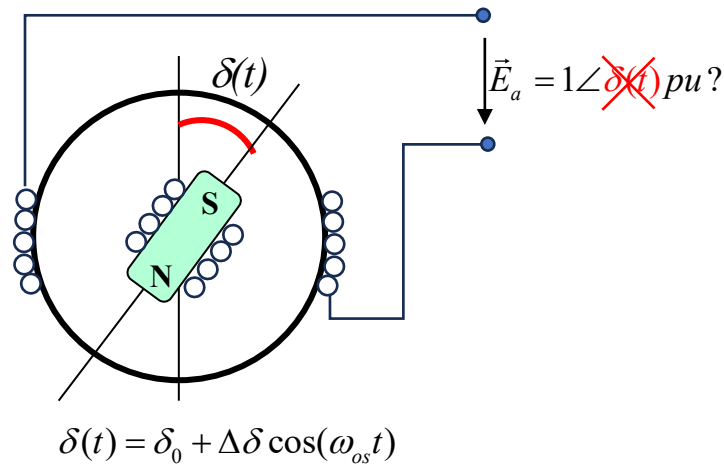
# 1D: Why there are interharmonics in a power system?

## Case 1: Natural frequencies of a circuit



- Any circuit has natural frequencies
- They are associated with the modes (or eigenvalues) of the circuit
- These frequencies are the interharmonics we are talking about here

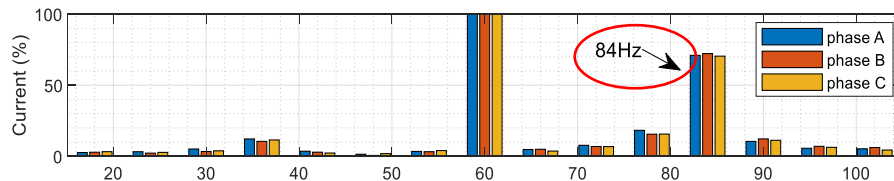
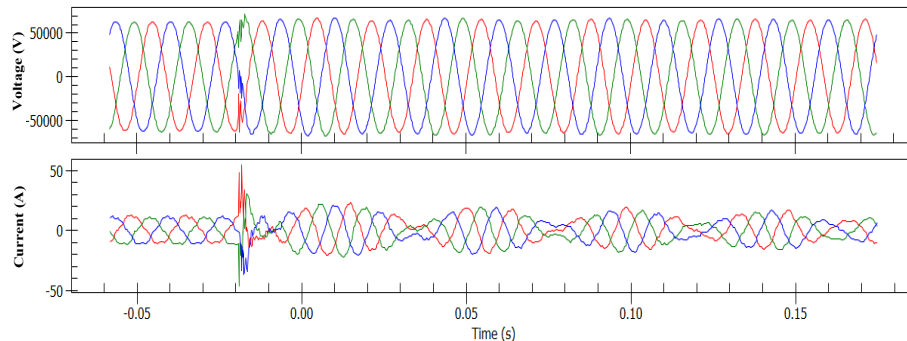
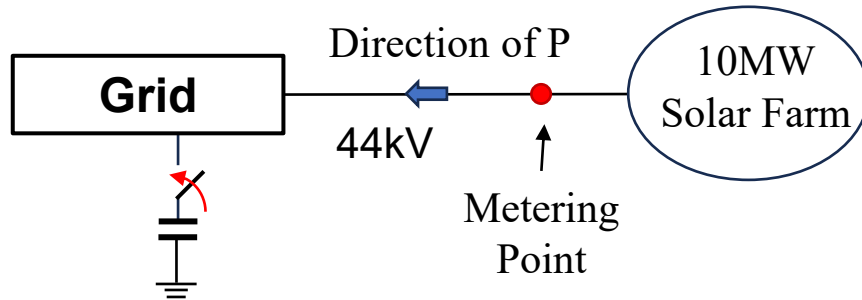
## Case 2: Synch. generator rotor oscillation



**Traditional teaching: Rotor oscillation produces a oscillating voltage phase angle**

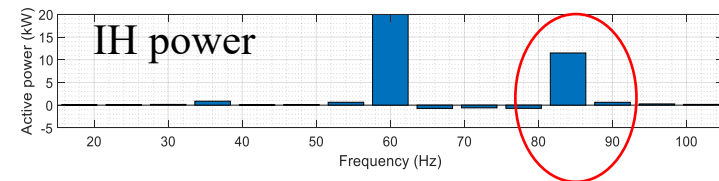
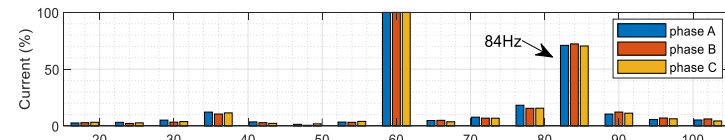
**Real phenomenon: Rotor oscillation produces interharmonic voltages at the SG terminal**

# 1E: Example case and application to source location



## Where is the source of oscillation?

- **Interharmonic power is needed to drive the interharmonics (i.e. phasor oscillation) in a system**
- **Therefore, interharmonic power producers are the sources causing oscillations**





## **1F. Waveform data versus phasor data**

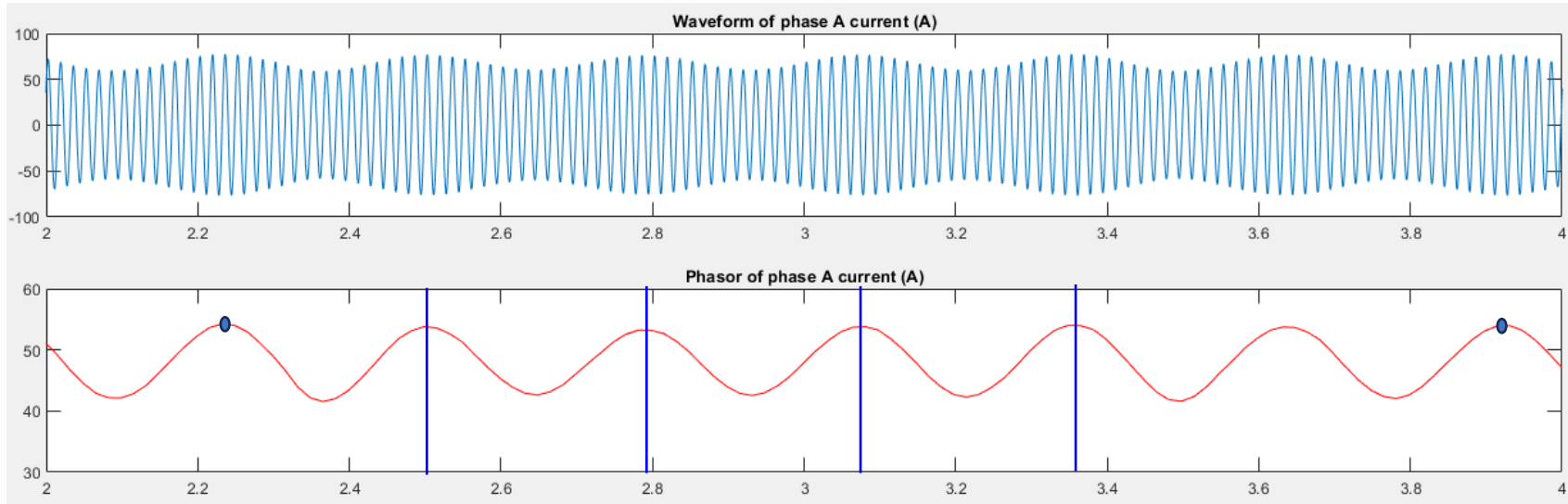
- **The materials presented so far are based on waveform data;**
- **Although there is a growing recognition of the value of waveform data nowadays, its availability for oscillation monitoring remains limited,**
- **On the other hand, PMU-based synchrophasor data is widely available,**
- **Can we extract interharmonic information from the PMU data?**
- **Can we use the results to develop oscillation source locators (without relying on waveform data)?**

## **2. Extracting interharmonics from phasor or synchrophasor data**

## 2A. How to extract interharmonics from phasor data?

**Step 1: Determining the oscillation frequency ( $f_{os}$ ) using **current** phasor data**

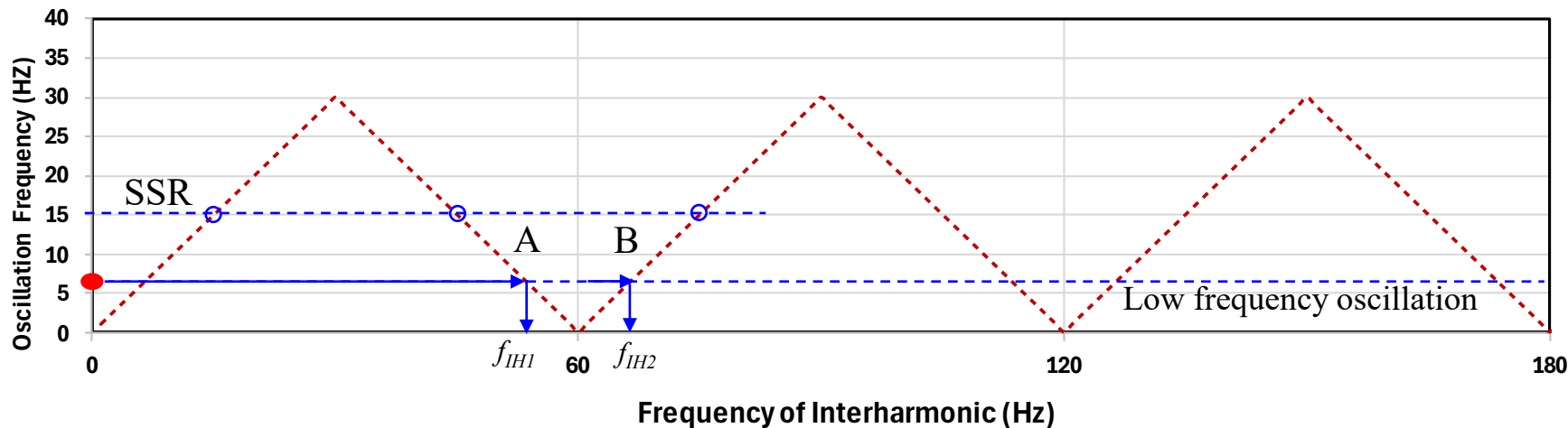
- FFT analysis on the phasor (magnitude) data
- Just counting the times between the peaks (=period of oscillation), average them



Field measured wind farm data

## 2B. How to extract interharmonics from phasor data?

### Step 2: Estimate the interharmonic frequency $f_{IH}$



- There are multiple possible  $f_{IH}$  frequencies
- Based on practical experiences, it is reasonable to assume the likely frequencies are at A and B for common low frequency oscillation events

$$f_{IH1} = f_1 - f_{os} = 60 - f_{os} \quad f_{IH2} = f_1 + f_{os} = 60 + f_{os}$$

- The algorithm (shown next) is not limited to A and B

## 2B. How to extract interharmonics from phasor data?

### Step 3: Establish models of interharmonic estimation

Waveform model:

$$\begin{aligned} v(t) &= \sqrt{2}V_1 \cos(\omega_1 t + \delta) + \overset{\text{IH1}}{\downarrow} \sqrt{2}V_2 \cos(\omega_2 t + \delta_2) + \overset{\text{IH2}}{\downarrow} \sqrt{2}V_3 \cos(\omega_3 t + \delta_3) \\ &= \sqrt{2}[X_1 \cos(\omega_1 t) - Y_1 \sin(\omega_1 t) + X_2 \cos(\omega_2 t) - Y_2 \sin(\omega_2 t) + X_3 \cos(\omega_3 t) - Y_3 \sin(\omega_3 t)] \end{aligned}$$

$$\omega_2 = \omega_{IH1}$$

$$\omega_3 = \omega_{IH2}$$

$$X_i = V_i \cos(\delta_i)$$

$$Y_i = V_i \sin(\delta_i)$$

**The goal is to find  $X_i$  and  $Y_i$**

Establishing the relationship with phasor results (for the  $k^{\text{th}}$  cycle):

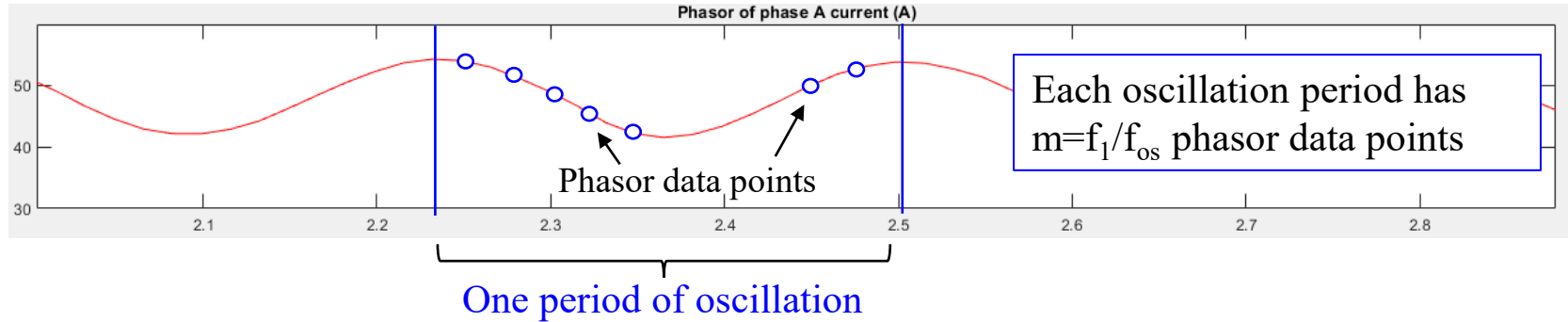
$$\vec{V}_{phasor}(k) = \frac{\sqrt{2}}{T_1} \int_{(k-1)T_1}^{kT_1} v(t) e^{-j2\pi f_0 t} dt = X_1 W_{1\cos-k} + Y_1 W_{1\sin-k} + X_2 W_{2\cos-k} + Y_2 W_{2\sin-k} + X_3 W_{3\cos-k} + Y_3 W_{3\sin-k}$$

$W_{i\cos-k}$  and  $W_{i\sin-k}$  are coefficients that can be calculated. One example formula is

$$W_{1\cos-k} = \frac{2\Delta T}{T_1} \sum_{k=1}^{k_2} \cos[\omega_1(k + K_{start})\Delta t] e^{-j\omega_0(k-1)\Delta T}$$

## 2B. How to extract interharmonics from phasor data?

**Step 4: Estimate  $X_i$  and  $Y_i$  values ( $i=1,2,3$ ) for each oscillation period**



$$\vec{V}_{phasor}(1) = X_1 W_{1\cos-1} + Y_1 W_{1\sin-1} + X_2 W_{2\cos-1} + Y_2 W_{2\sin-1} + X_3 W_{3\cos-1} + Y_3 W_{3\sin-1}$$

....

$$\vec{V}_{phasor}(k) = X_1 W_{1\cos-k} + Y_1 W_{1\sin-k} + X_2 W_{2\cos-k} + Y_2 W_{2\sin-k} + X_3 W_{3\cos-k} + Y_3 W_{3\sin-k}$$

....

$$\vec{V}_{phasor}(m) = X_1 W_{1\cos-m} + Y_1 W_{1\sin-m} + X_2 W_{2\cos-m} + Y_2 W_{2\sin-m} + X_3 W_{3\cos-m} + Y_3 W_{3\sin-m}$$

- $X_1, Y_1, X_2, Y_2, X_3, Y_3$  can be solved from the above equation using least square fitting method
- “m” must be greater than 6 for estimation to work

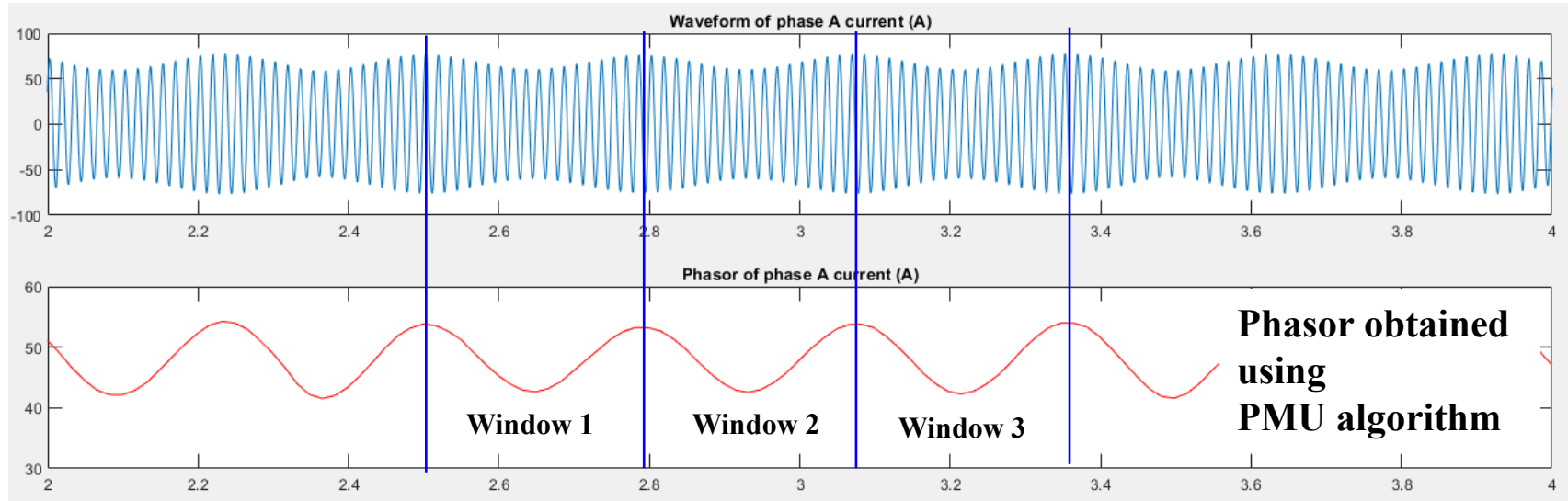
## 2C. Summary of steps

1. Take in the phasor data of the oscillation event to be processed
2. Estimate  $f_{os}$  using the oscillation-containing segment of the current data
3. Estimate the interharmonic frequencies  $f_{ih1}$  and  $f_{ih2}$
4. Step through each oscillation period
  - Compute W coefficients for the period
  - Calculate  $X_i$  and  $Y_i$  for the period (i.e. 60Hz and interharmonic magnitudes and phases)
  - Calculate interharmonic powers for the period
5. Check the signs of IH powers. If needed, compare the IH powers of different sites
6. Draw conclusions on 1) which sites are sources and 2) their respective contributions based on the magnitudes of IH powers

### **3. Case Studies and Sensitivity Analysis**



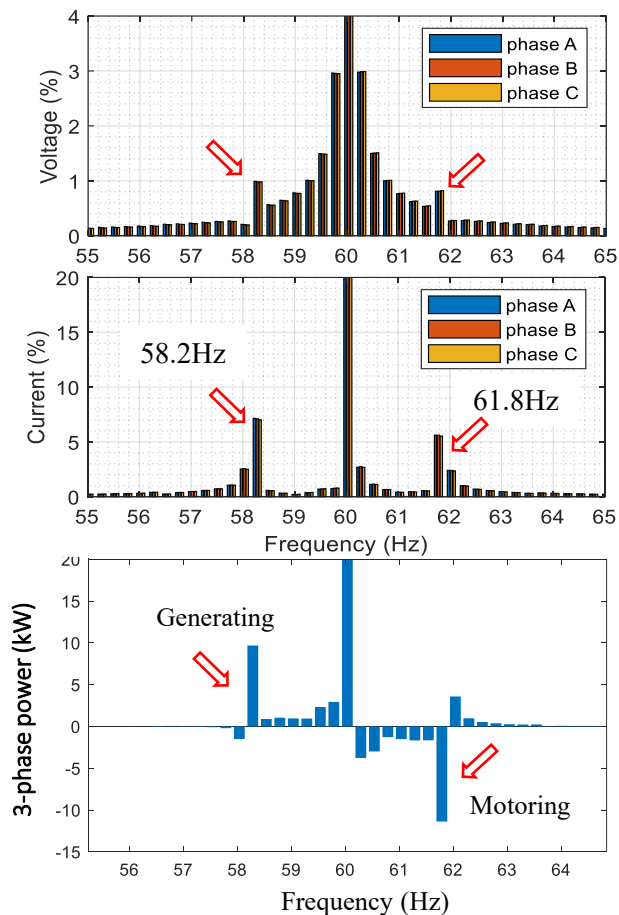
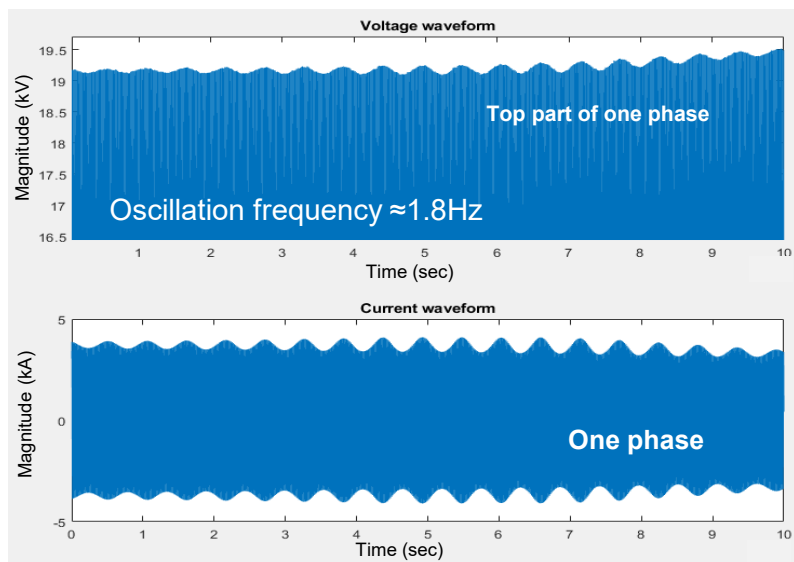
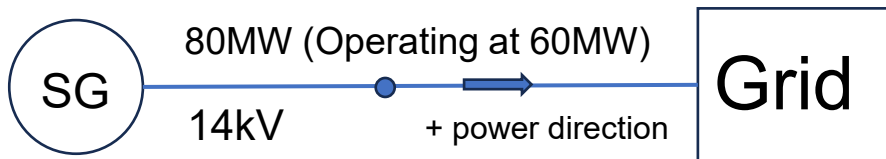
### 3. How to evaluate the IH extraction method?



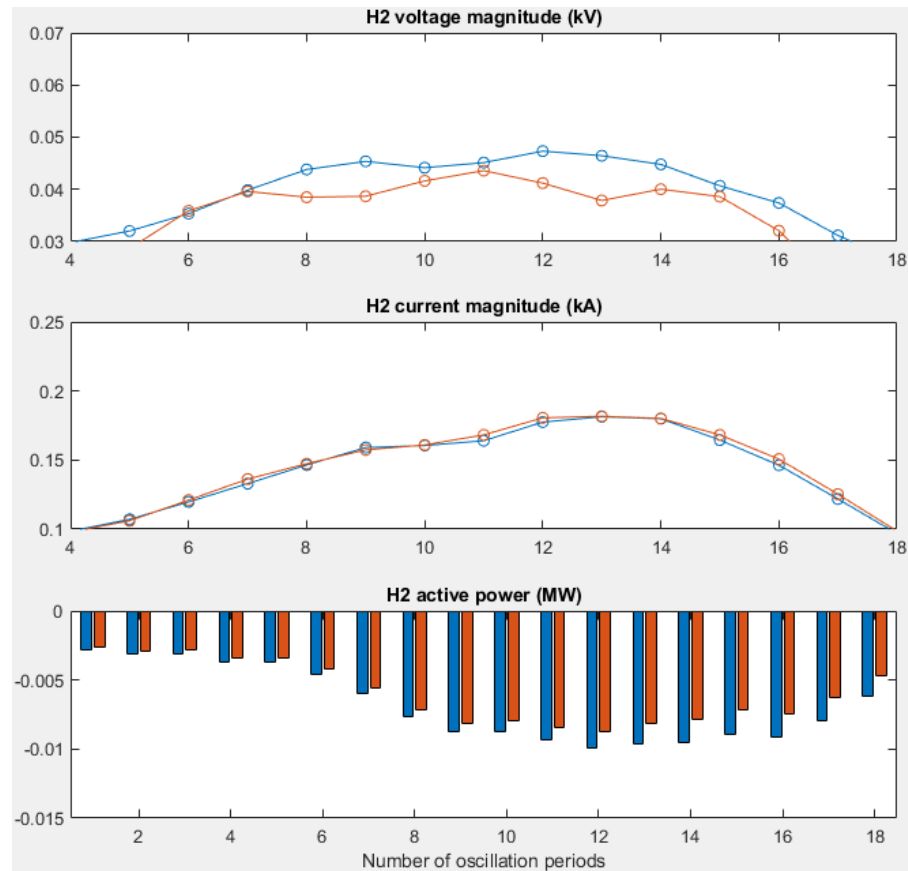
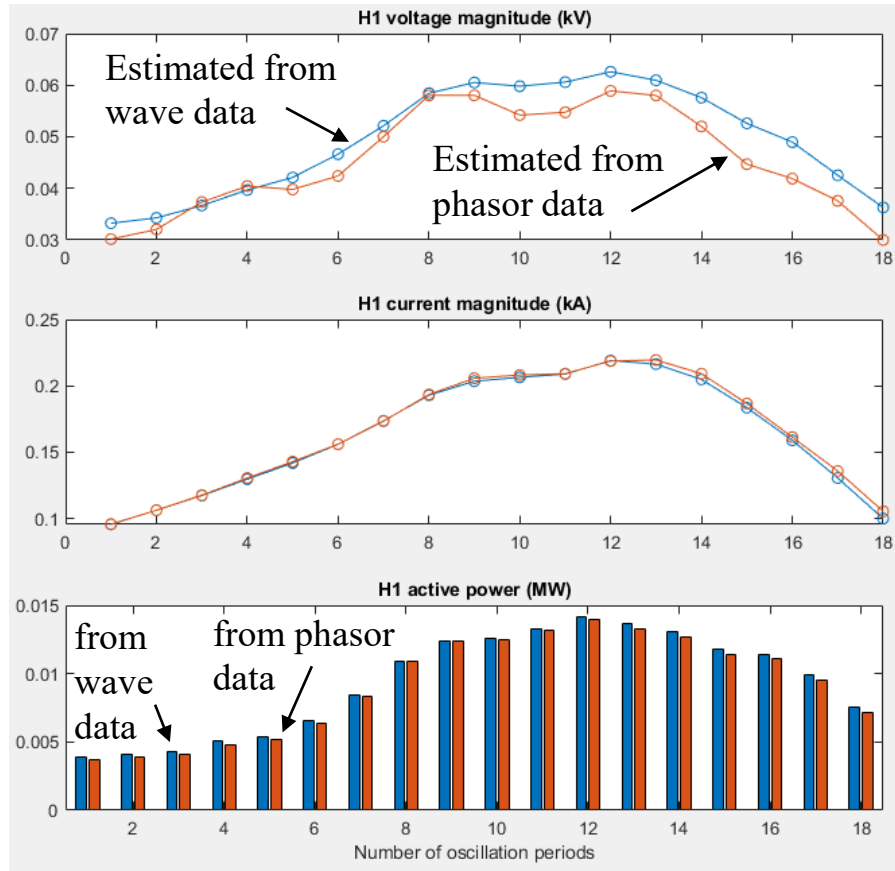
**Use the same windows (i.e. periods of oscillation):**

1. Extract interharmonics from the waveform data using a simple FFT method,
2. Extract interharmonics from the phasor result of the same period using the proposed method
3. Compare the interharmonic results obtained from 1 and 2 over multiple periods

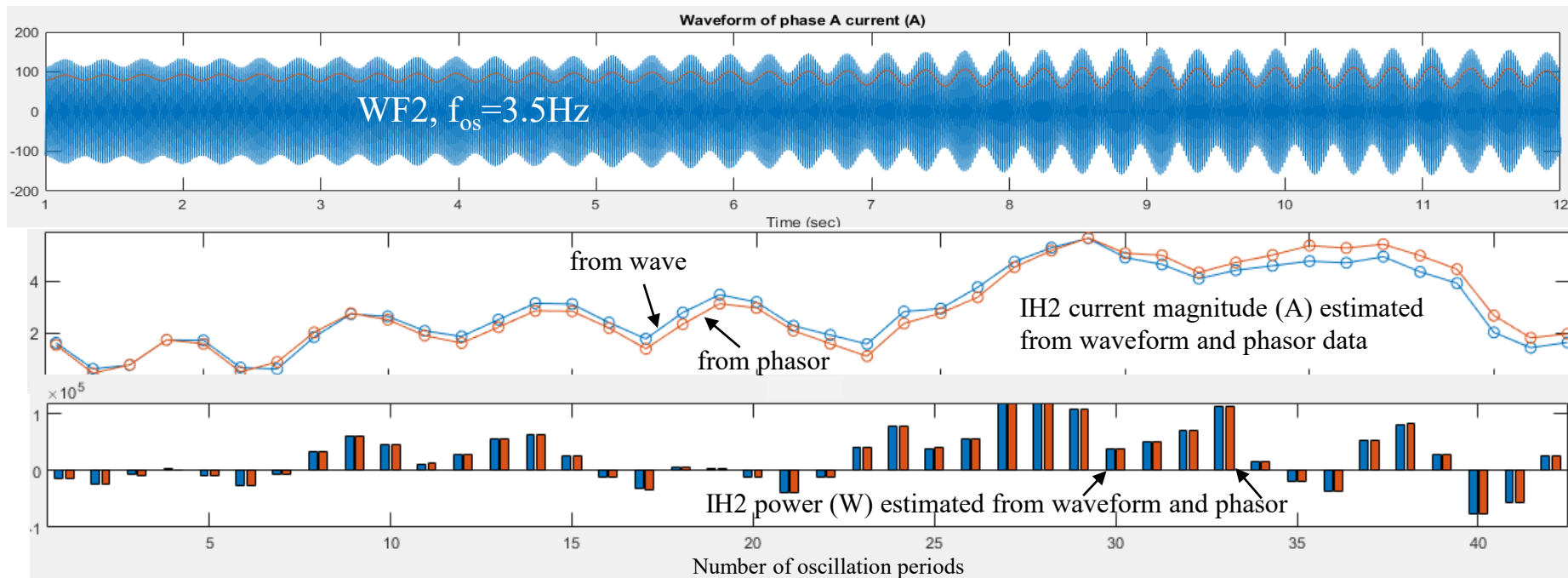
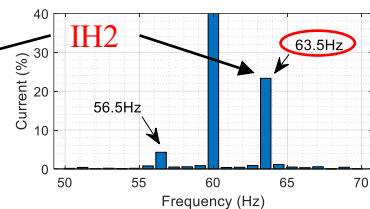
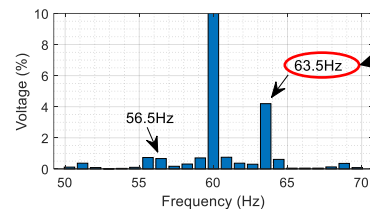
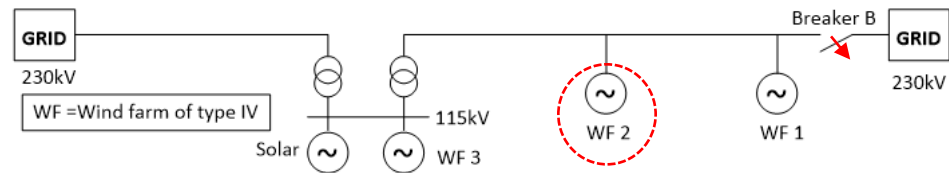
# 3A. Synchronous Generator Oscillation (field data)



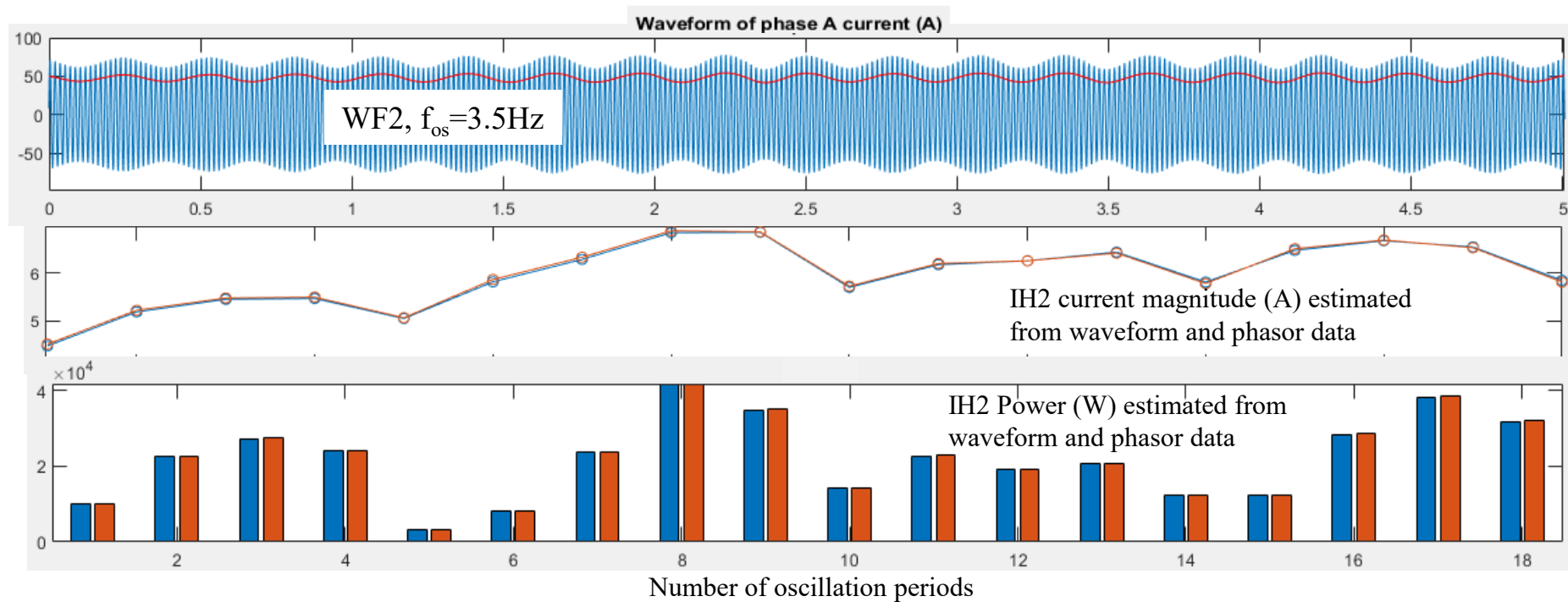
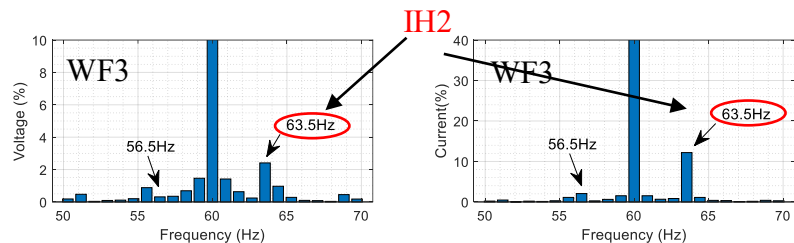
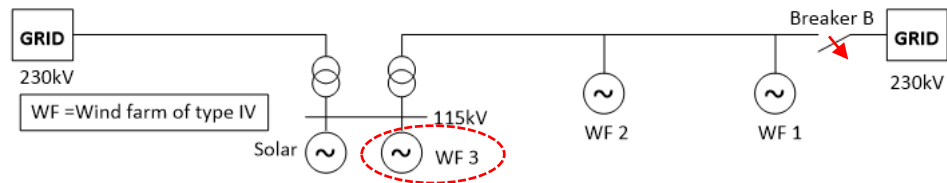
# 3A. Synchronous Generator Oscillation (field data)



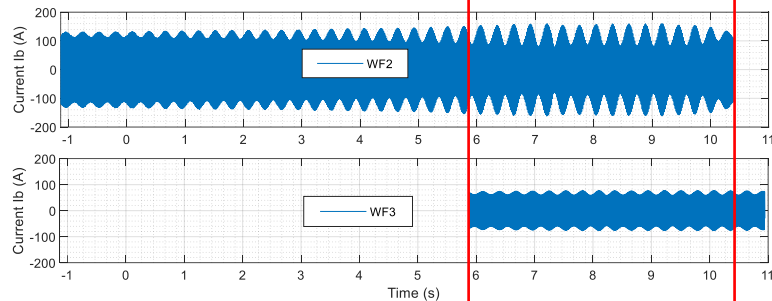
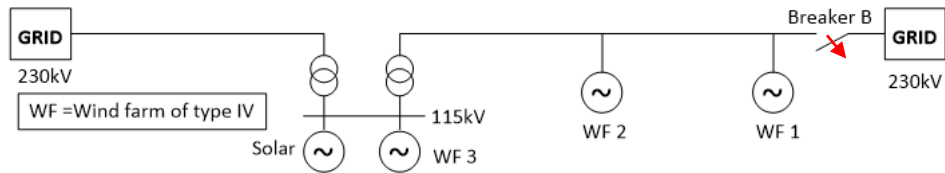
# 3B. Wind Farm 2 (WF2) Oscillation (field data)



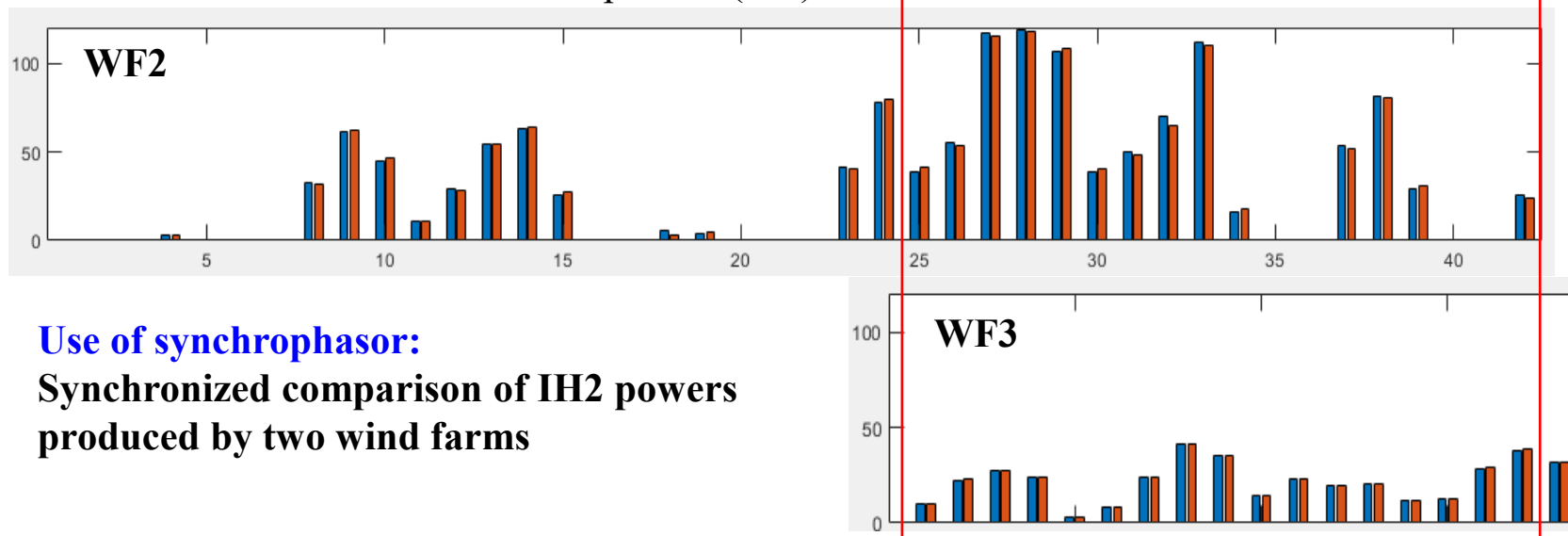
# 3B. Wind Farm 3 (WF3) Oscillation (field data)



### 3B. Comparing Wind Farms 2 and 3 (field data)



IH2 powers (kW)

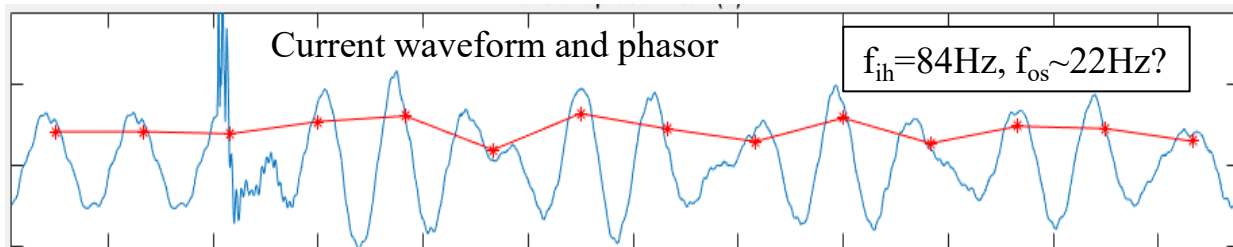


**Use of synchrophasor:**

**Synchronized comparison of IH2 powers produced by two wind farms**

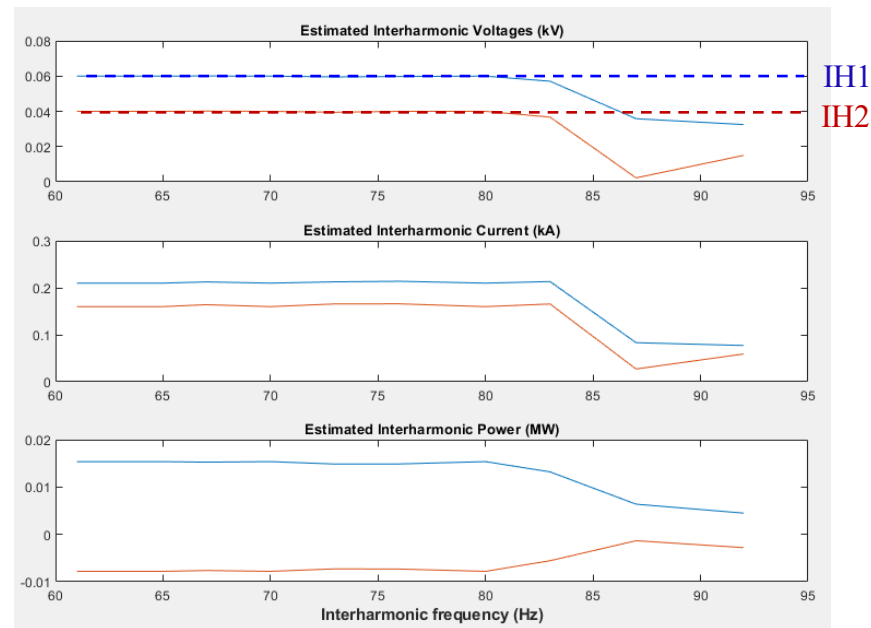
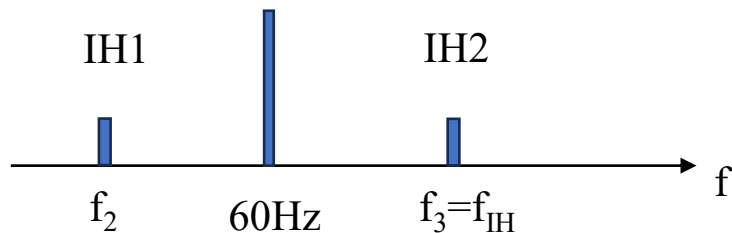
# 3C – Solar Farm (field data)

- Phasor oscillation pattern is not clear
- $m=60/f_{os} \approx 2 < 6 \Rightarrow$  cannot estimate IH.

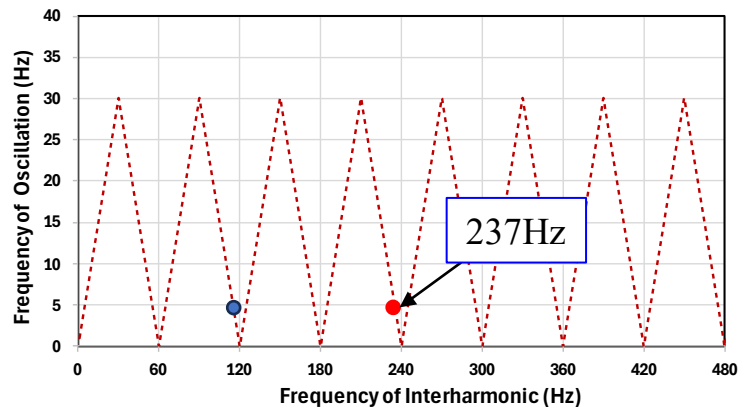
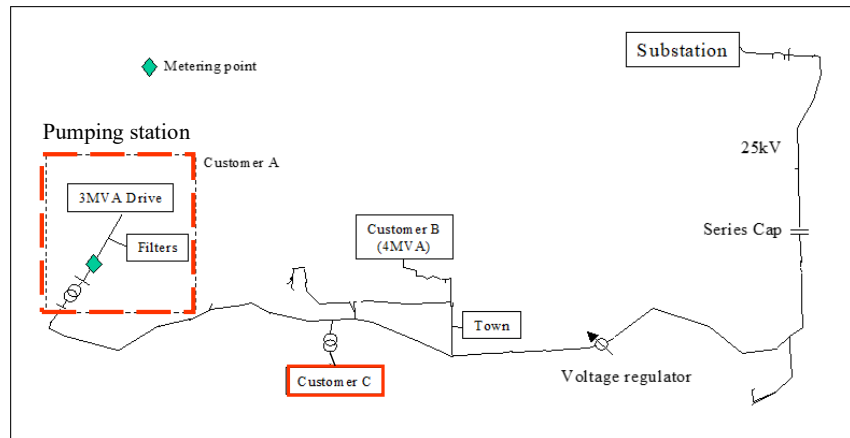


## Sensitivity study:

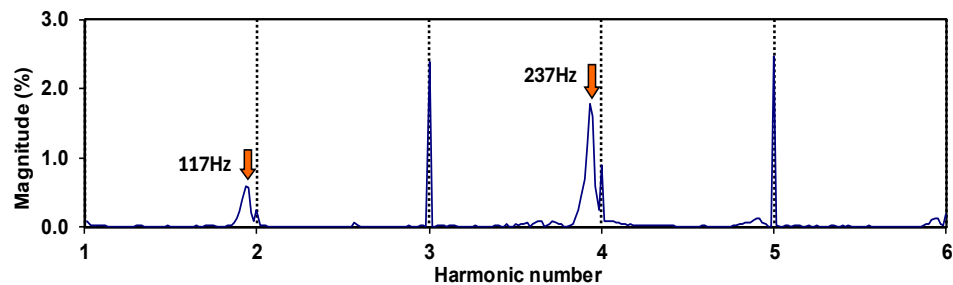
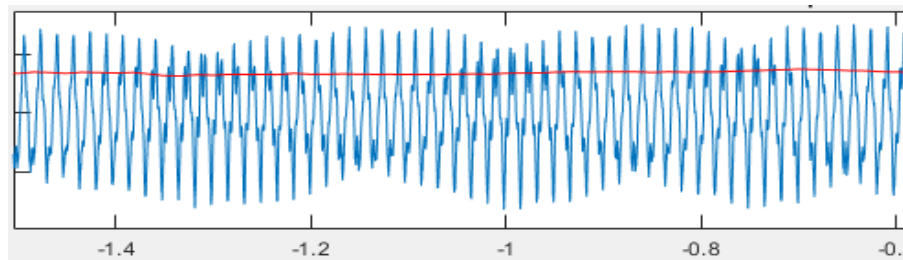
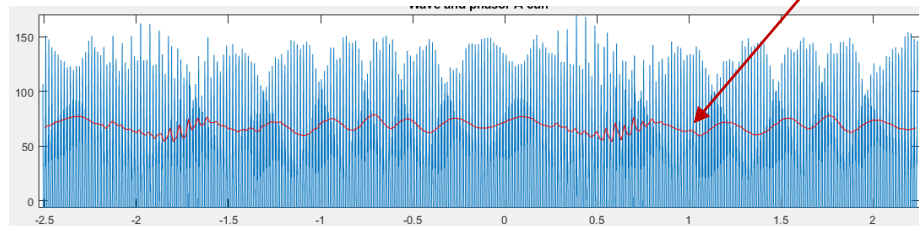
$$v(t) = \sqrt{2}V_1 \cos(\omega_1 t + \delta) + \sqrt{2}V_2 \cos(\omega_2 t + \delta_2) + \sqrt{2}V_3 \cos(\omega_3 t + \delta_3)$$



# 3D. Industrial plant (field data)



Current waveform





### 3E. Sensitivity Study Results (Model Data)

Study Model: 
$$v(t) = \sqrt{2}V_1 \cos(\omega_1 t + \delta) + \sqrt{2}V_2 \cos(\omega_2 t + \delta_2) + \sqrt{2}V_3 \cos(\omega_3 t + \delta_3)$$

**It is found that the IH estimation results are**

**Not sensitive to:**

- PMU sampling rate (3~64 samples/cycle). But the rate is needed to calculate W coefficients
- One or two interharmonic components

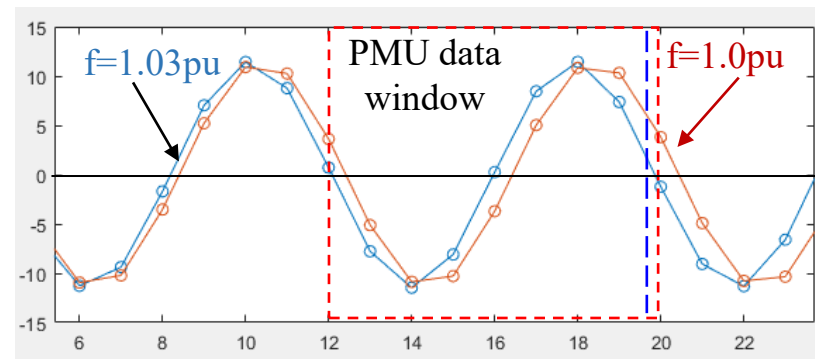
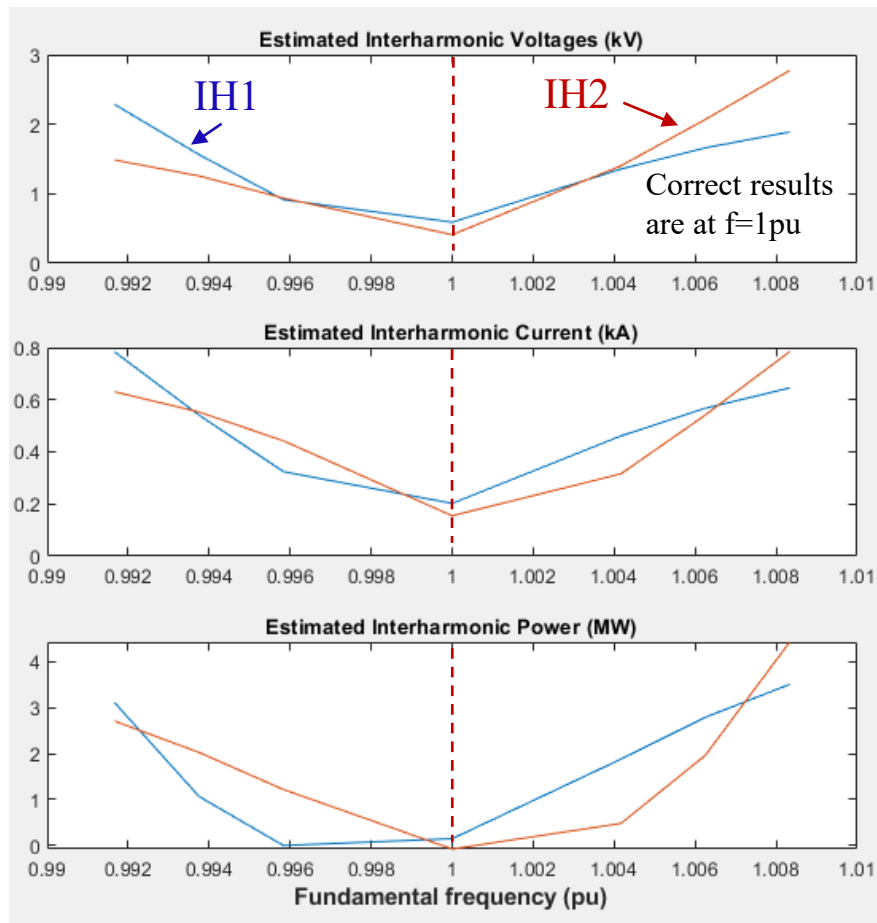
**Mildly sensitive to:**

- The weighting function used for phasor calculation 
$$\vec{V}_{phasor}(k) = \frac{\sqrt{2}}{T_1} \int_{(k-1)T_1}^{kT_1} w(t)v(t)e^{-j2\pi f_0 t} dt$$
- Accurate estimation of oscillation frequency (which will affect W coefficients) if  $f_{os} < 5\text{Hz}$

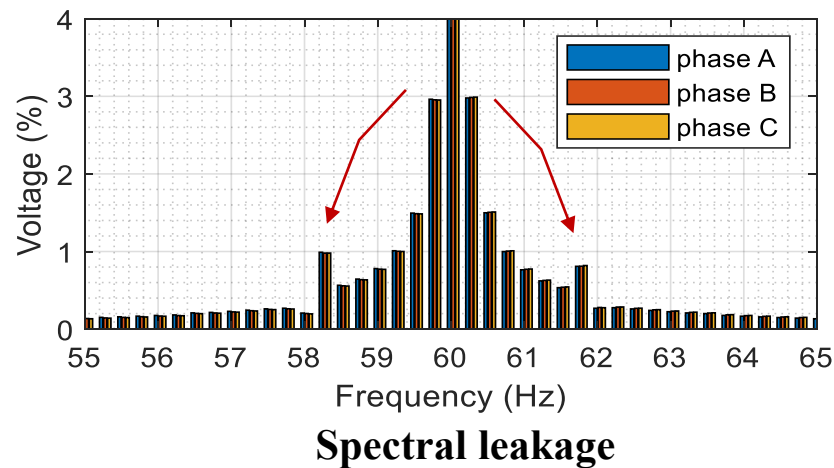
**Sensitive to:**

- Interharmonic frequencies if they are far different from 60Hz (shown in the solar farm slide)
- Deviation of frequency from 60Hz (shown next)

### 3E. Sensitivity Study Results (Model Data) – Impact of system frequency



Impact of PMU window



## 4. Conclusions and takeaways

- **Phasor oscillation is caused by interharmonics. Interharmonic data is needed to diagnose oscillations;**
- **It is possible to extract reliable interharmonic data from PMU data if**
  - 1) The oscillation frequency is low such as less than 5Hz, which is common
  - 2) A consistent sinusoidal oscillation pattern exists for some periods;
  - 3) Frequency deviation is not significant;
- **The algorithm for IH estimation is simple to understand and easy to implement. One immediate application is oscillation source location;**
- **We need to be careful when applying the method to IBR data due to possible involvement of high frequency interharmonic components.**

# Thank you

## I welcome any questions and comments you may have

Information on interharmonics causing oscillation can be found from the following paper:

W. Xu, J. Yong, H. J. Marquez and C. Li, "Interharmonic Power – A New Concept for Power System Oscillation Source Location," in IEEE Transactions on Power Systems, doi: 10.1109/TPWRS.2025.3535863.